

Rapael System

■ YRP (Roll ϕ $\hat{x}_B$, Pitch θ $\hat{y}_B$, Yaw ψ $\hat{z}_B$, Z_{Forearm} X_{Wrist} $Y_{\text{Deviation}}$)

$$\begin{pmatrix} \cos[\psi] & -\sin[\psi] & 0 \\ \sin[\psi] & \cos[\psi] & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos[\phi] & -\sin[\phi] \\ 0 & \sin[\phi] & \cos[\phi] \end{pmatrix} \cdot \begin{pmatrix} \cos[\theta] & 0 & \sin[\theta] \\ 0 & 1 & 0 \\ -\sin[\theta] & 0 & \cos[\theta] \end{pmatrix} // \text{MatrixForm}$$

$$\begin{pmatrix} \cos[\theta] \cos[\psi] - \sin[\theta] \sin[\phi] \sin[\psi] & -\cos[\phi] \sin[\psi] & \cos[\psi] \sin[\theta] + \cos[\theta] \sin[\phi] \sin[\psi] \\ \cos[\psi] \sin[\theta] \sin[\phi] + \cos[\theta] \sin[\psi] & \cos[\phi] \cos[\psi] & -\cos[\theta] \cos[\psi] \sin[\phi] + \sin[\theta] \sin[\psi] \\ -\cos[\phi] \sin[\theta] & \sin[\phi] & \cos[\theta] \cos[\phi] \end{pmatrix}$$

$$\hat{\mathbf{B}}\mathbf{R} = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} = \begin{pmatrix} 2q_1^2 - 1 + 2q_2^2 & 2(q_2q_3 + q_1q_4) & 2(q_2q_4 - q_1q_3) \\ 2(q_2q_3 - q_1q_4) & 2q_1^2 - 1 + 2q_3^2 & 2(q_3q_4 + q_1q_2) \\ 2(q_2q_4 + q_1q_3) & 2(q_3q_4 - q_1q_2) & 2q_1^2 - 1 + 2q_4^2 \end{pmatrix} = \begin{pmatrix} 1 - 2(q_3^2 + 2q_4^2) & 2(q_2q_3 + q_1q_4) & 2(q_2q_4 - q_1q_3) \\ 2(q_2q_3 - q_1q_4) & 1 - 2(q_2^2 + 2q_4^2) & 2(q_3q_4 + q_1q_2) \\ 2(q_2q_4 + q_1q_3) & 2(q_3q_4 - q_1q_2) & 1 - 2(q_2^2 + 2q_3^2) \end{pmatrix}$$

$$(q_1 \quad -q_2 \quad -q_3 \quad -q_4)$$

$$\hat{\mathbf{B}}\mathbf{R} = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} = \begin{pmatrix} 1 - 2(q_3^2 + 2q_4^2) & 2(q_2q_3 - q_1q_4) & 2(q_2q_4 + q_1q_3) \\ 2(q_2q_3 + q_1q_4) & 1 - 2(q_2^2 + 2q_4^2) & 2(q_3q_4 - q_1q_2) \\ 2(q_2q_4 - q_1q_3) & 2(q_3q_4 + q_1q_2) & 1 - 2(q_2^2 + 2q_3^2) \end{pmatrix}$$

$$\phi = \text{ArcSin}[\hat{\mathbf{B}}\mathbf{R}_{32}] = \text{ArcTan}[\sqrt{\hat{\mathbf{B}}\mathbf{R}_{31}^2 + \hat{\mathbf{B}}\mathbf{R}_{33}^2}, \hat{\mathbf{B}}\mathbf{R}_{32}] = \text{ArcSin}[2(q_3q_4 - q_1q_2)]$$

$$\theta = \text{ArcTan}[\hat{\mathbf{B}}\mathbf{R}_{33}, -\hat{\mathbf{B}}\mathbf{R}_{31}] = \text{ArcTan}[2q_1^2 - 1 + 2q_4^2, -2(q_2q_4 + q_1q_3)]$$

$$\psi = \text{ArcTan}[\hat{\mathbf{B}}\mathbf{R}_{22}, -\hat{\mathbf{B}}\mathbf{R}_{12}] = \text{ArcTan}[2q_1^2 - 1 + 2q_3^2, -2(q_2q_3 + q_1q_4)]$$

Sensor Euler Angle (Roll ϕ $\hat{x}_B$, Pitch θ $\hat{y}_B$, Yaw ψ $\hat{z}_B$, ZYX) $\rightarrow$ Quaternion $\hat{\mathbf{B}}\hat{\mathbf{q}}$

$$\hat{\mathbf{B}}\mathbf{R} = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} = \begin{pmatrix} 2q_1^2 - 1 + 2q_2^2 & 2(q_2q_3 + q_1q_4) & 2(q_2q_4 - q_1q_3) \\ 2(q_2q_3 - q_1q_4) & 2q_1^2 - 1 + 2q_3^2 & 2(q_3q_4 + q_1q_2) \\ 2(q_2q_4 + q_1q_3) & 2(q_3q_4 - q_1q_2) & 2q_1^2 - 1 + 2q_4^2 \end{pmatrix} = \begin{pmatrix} 1 - 2(q_y^2 + q_z^2) & 2(q_xq_y - q_wq_z) & 2(q_xq_z + q_wq_y) \\ 2(q_xq_y + q_wq_z) & 1 - 2(q_x^2 + q_z^2) & 2(q_yq_z - q_wq_x) \\ 2(q_xq_z - q_wq_y) & 2(q_yq_z + q_wq_x) & 1 - 2(q_x^2 + q_y^2) \end{pmatrix}$$

Forearm, Deviation, Wrist

■ YPR $\rightarrow$ R축 회전, z축(Y) 회전

$$\begin{pmatrix} \cos[\psi] & -\sin[\psi] & 0 \\ \sin[\psi] & \cos[\psi] & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos[\theta] & 0 & \sin[\theta] \\ 0 & 1 & 0 \\ -\sin[\theta] & 0 & \cos[\theta] \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos[\phi] & -\sin[\phi] \\ 0 & \sin[\phi] & \cos[\phi] \end{pmatrix} // \text{MatrixForm}$$

$$\begin{pmatrix} \cos[\theta] \cos[\psi] & \cos[\psi] \sin[\theta] \sin[\phi] - \cos[\phi] \sin[\psi] & \cos[\phi] \cos[\psi] \sin[\theta] + \sin[\phi] \sin[\psi] \\ \cos[\theta] \sin[\psi] & \cos[\phi] \cos[\psi] + \sin[\theta] \sin[\phi] \sin[\psi] & -\cos[\psi] \sin[\phi] + \cos[\phi] \sin[\theta] \sin[\psi] \\ -\sin[\theta] & \cos[\theta] \sin[\phi] & \cos[\theta] \cos[\phi] \end{pmatrix}$$

$$\text{R축 회전} = \text{ArcTan}[r_{33}, r_{32}]$$

$$\text{z축 (Y) 회전} = \text{ArcTan}[r_{11}, r_{21}]$$

■ RYP $\rightarrow$ P축 회전, x축(R) 회전

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos[\phi] & -\sin[\phi] \\ 0 & \sin[\phi] & \cos[\phi] \end{pmatrix} \cdot \begin{pmatrix} \cos[\psi] & -\sin[\psi] & 0 \\ \sin[\psi] & \cos[\psi] & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos[\theta] & 0 & \sin[\theta] \\ 0 & 1 & 0 \\ -\sin[\theta] & 0 & \cos[\theta] \end{pmatrix} // \text{MatrixForm}$$

$$\begin{pmatrix} \cos[\theta] \cos[\psi] & -\sin[\psi] & \cos[\psi] \sin[\theta] \\ \sin[\theta] \sin[\phi] + \cos[\theta] \cos[\phi] \sin[\psi] & \cos[\phi] \cos[\psi] & -\cos[\theta] \sin[\phi] + \cos[\phi] \sin[\theta] \sin[\psi] \\ -\cos[\phi] \sin[\theta] + \cos[\theta] \sin[\phi] \sin[\psi] & \cos[\psi] \sin[\phi] & \cos[\theta] \cos[\phi] + \sin[\theta] \sin[\phi] \sin[\psi] \end{pmatrix}$$

$$\text{P축 회전} = \text{ArcTan}[r_{11}, r_{13}]$$

$$\text{x축 (R) 회전} = \text{ArcTan}[r_{22}, r_{32}]$$

■ PRY $\rightarrow$ Y축 회전, y축(P) 회전

$$\begin{pmatrix} \cos[\theta] & 0 & \sin[\theta] \\ 0 & 1 & 0 \\ -\sin[\theta] & 0 & \cos[\theta] \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos[\phi] & -\sin[\phi] \\ 0 & \sin[\phi] & \cos[\phi] \end{pmatrix} \cdot \begin{pmatrix} \cos[\psi] & -\sin[\psi] & 0 \\ \sin[\psi] & \cos[\psi] & 0 \\ 0 & 0 & 1 \end{pmatrix} // \text{MatrixForm}$$

$$\begin{pmatrix} \cos[\theta] \cos[\psi] + \sin[\theta] \sin[\phi] \sin[\psi] & \cos[\psi] \sin[\theta] \sin[\phi] - \cos[\theta] \sin[\psi] & \cos[\phi] \sin[\theta] \\ \cos[\phi] \sin[\psi] & \cos[\phi] \cos[\psi] & -\sin[\phi] \\ -\cos[\psi] \sin[\theta] + \cos[\theta] \sin[\phi] \sin[\psi] & \cos[\theta] \cos[\psi] \sin[\phi] + \sin[\theta] \sin[\psi] & \cos[\theta] \cos[\phi] \end{pmatrix}$$

$$\text{y축 (P) 회전} = \text{ArcTan}[r_{33}, r_{13}]$$

$$\text{Y축 회전 Forearm} = \text{ArcTan}[r_{22}, r_{21}]$$

■ **PYR → R축 회전, y축(P) 회전**

$$\begin{pmatrix} \text{Cos}[\theta] & 0 & \text{Sin}[\theta] \\ 0 & 1 & 0 \\ -\text{Sin}[\theta] & 0 & \text{Cos}[\theta] \end{pmatrix} \cdot \begin{pmatrix} \text{Cos}[\psi] & -\text{Sin}[\psi] & 0 \\ \text{Sin}[\psi] & \text{Cos}[\psi] & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \text{Cos}[\phi] & -\text{Sin}[\phi] \\ 0 & \text{Sin}[\phi] & \text{Cos}[\phi] \end{pmatrix} // \text{MatrixForm}$$
$$\begin{pmatrix} \text{Cos}[\theta] \text{Cos}[\psi] & \text{Sin}[\theta] \text{Sin}[\phi] - \text{Cos}[\theta] \text{Cos}[\phi] \text{Sin}[\psi] & \text{Cos}[\phi] \text{Sin}[\theta] + \text{Cos}[\theta] \text{Sin}[\phi] \text{Sin}[\psi] \\ \text{Sin}[\psi] & \text{Cos}[\phi] \text{Cos}[\psi] & -\text{Cos}[\psi] \text{Sin}[\phi] \\ -\text{Cos}[\psi] \text{Sin}[\theta] & \text{Cos}[\theta] \text{Sin}[\phi] + \text{Cos}[\phi] \text{Sin}[\theta] \text{Sin}[\psi] & \text{Cos}[\theta] \text{Cos}[\phi] - \text{Sin}[\theta] \text{Sin}[\phi] \text{Sin}[\psi] \end{pmatrix}$$

R축 회전 = ArcTan[r₂₂, -r₂₃]
y축 (P) 회전 = ArcTan[r₁₁, -r₃₁]

■ **YRP → P축 회전, z축(Y) 회전**

$$\begin{pmatrix} \text{Cos}[\psi] & -\text{Sin}[\psi] & 0 \\ \text{Sin}[\psi] & \text{Cos}[\psi] & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \text{Cos}[\phi] & -\text{Sin}[\phi] \\ 0 & \text{Sin}[\phi] & \text{Cos}[\phi] \end{pmatrix} \cdot \begin{pmatrix} \text{Cos}[\theta] & 0 & \text{Sin}[\theta] \\ 0 & 1 & 0 \\ -\text{Sin}[\theta] & 0 & \text{Cos}[\theta] \end{pmatrix} // \text{MatrixForm}$$
$$\begin{pmatrix} \text{Cos}[\theta] \text{Cos}[\psi] - \text{Sin}[\theta] \text{Sin}[\phi] \text{Sin}[\psi] & -\text{Cos}[\phi] \text{Sin}[\psi] & \text{Cos}[\psi] \text{Sin}[\theta] + \text{Cos}[\theta] \text{Sin}[\phi] \text{Sin}[\psi] \\ \text{Cos}[\psi] \text{Sin}[\theta] \text{Sin}[\phi] + \text{Cos}[\theta] \text{Sin}[\psi] & \text{Cos}[\phi] \text{Cos}[\psi] & -\text{Cos}[\theta] \text{Cos}[\psi] \text{Sin}[\phi] + \text{Sin}[\theta] \text{Sin}[\psi] \\ -\text{Cos}[\phi] \text{Sin}[\theta] & \text{Sin}[\phi] & \text{Cos}[\theta] \text{Cos}[\phi] \end{pmatrix}$$

P축 회전 Deviation = ArcTan[r₃₃, -r₃₁]
z축 (Y) 회전 = ArcTan[r₂₂, -r₁₂]

■ **RPY → Y축 회전, x축(R) 회전**

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \text{Cos}[\phi] & -\text{Sin}[\phi] \\ 0 & \text{Sin}[\phi] & \text{Cos}[\phi] \end{pmatrix} \cdot \begin{pmatrix} \text{Cos}[\theta] & 0 & \text{Sin}[\theta] \\ 0 & 1 & 0 \\ -\text{Sin}[\theta] & 0 & \text{Cos}[\theta] \end{pmatrix} \cdot \begin{pmatrix} \text{Cos}[\psi] & -\text{Sin}[\psi] & 0 \\ \text{Sin}[\psi] & \text{Cos}[\psi] & 0 \\ 0 & 0 & 1 \end{pmatrix} // \text{MatrixForm}$$
$$\begin{pmatrix} \text{Cos}[\theta] \text{Cos}[\psi] & -\text{Cos}[\theta] \text{Sin}[\psi] & \text{Sin}[\theta] \\ \text{Cos}[\psi] \text{Sin}[\theta] \text{Sin}[\phi] + \text{Cos}[\phi] \text{Sin}[\psi] & \text{Cos}[\phi] \text{Cos}[\psi] - \text{Sin}[\theta] \text{Sin}[\phi] \text{Sin}[\psi] & -\text{Cos}[\theta] \text{Sin}[\phi] \\ -\text{Cos}[\phi] \text{Cos}[\psi] \text{Sin}[\theta] + \text{Sin}[\phi] \text{Sin}[\psi] & \text{Cos}[\psi] \text{Sin}[\phi] + \text{Cos}[\phi] \text{Sin}[\theta] \text{Sin}[\psi] & \text{Cos}[\theta] \text{Cos}[\phi] \end{pmatrix}$$

x축 (R) 회전 Wrist = ArcTan[r₃₃, -r₂₃]
y축 회전 = ArcTan[r₁₁, -r₁₂]

YRP (Roll $\phi \hat{x}_B$, Pitch $\theta \hat{y}_B$, Yaw $\psi \hat{z}_B$, Z_{Forearm} X_{Wrist} Y_{Deviation})

■ **Forearm**

y축 회전 = ArcTan[r₂₂, r₂₁]

■ **Deviation**

P축 회전 = ArcTan[r₃₃, -r₃₁]

■ **Wrist**

x축 (R) 회전 = ArcTan[r₃₃, -r₂₃]