

Magnetic sensor Soft/Hard Iron

Yoo, Kyunghwan

Least Squares LR (Linear Regression)

■ $y = w_0 + w_1 x$

■ 1

$$\sum_{i=1}^n y_i = \sum_{i=1}^n (w_0 + w_1 x_i)$$
$$\sum_{i=1}^n y_i x_i = \sum_{i=1}^n (w_0 x_i + w_1 x_i^2)$$

■ 2

$$\begin{pmatrix} 1 & \dots & 1 \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} 1 & \dots & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & x_1 \\ \vdots & \vdots \\ 1 & x_n \end{pmatrix} \cdot \begin{pmatrix} w_0 \\ w_1 \end{pmatrix}$$
$$\begin{pmatrix} x_1 & \dots & x_n \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} x_1 & \dots & x_n \end{pmatrix} \cdot \begin{pmatrix} 1 & x_1 \\ \vdots & \vdots \\ 1 & x_n \end{pmatrix} \cdot \begin{pmatrix} w_0 \\ w_1 \end{pmatrix}$$

■ 3

$$\begin{pmatrix} 1 & \dots & 1 \\ x_1 & \dots & x_n \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} 1 & \dots & 1 \\ x_1 & \dots & x_n \end{pmatrix} \cdot \begin{pmatrix} 1 & x_1 \\ \vdots & \vdots \\ 1 & x_n \end{pmatrix} \cdot \begin{pmatrix} w_0 \\ w_1 \end{pmatrix}$$

■ 4

$$X^T Y = X^T X W$$
$$(X^T X)^{-1} (X^T X) W = (X^T X)^{-1} (X^T Y)$$
$$W = (X^T X)^{-1} (X^T Y)$$

■ $y = w_0 + w_1 x1 + w_1 x2$

■ 1

$$\sum_{i=1}^n y_i = \sum_{i=1}^n (w_0 + w_1 (x1_i + x2_i))$$
$$\sum_{i=1}^n y_i (x1_i + x2_i) = \sum_{i=1}^n (w_0 (x1_i + x2_i) + w_1 (x1_i + x2_i)^2)$$

■ 2

$$\begin{pmatrix} 1 & \dots & 1 \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} 1 & \dots & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & x1_1 + x2_1 \\ \vdots & \vdots \\ 1 & x1_n + x2_n \end{pmatrix} \cdot \begin{pmatrix} w_0 \\ w_1 \end{pmatrix}$$
$$\begin{pmatrix} x1_1 + x2_1 & \dots & x1_n + x2_n \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} x1_1 + x2_1 & \dots & x1_n + x2_n \end{pmatrix} \cdot \begin{pmatrix} 1 & x1_1 + x2_1 \\ \vdots & \vdots \\ 1 & x1_n + x2_n \end{pmatrix} \cdot \begin{pmatrix} w_0 \\ w_1 \end{pmatrix}$$

■ 3

$$\begin{pmatrix} 1 & \dots & 1 \\ x1_1 + x2_1 & \dots & x1_n + x2_n \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} 1 & \dots & 1 \\ x1_1 + x2_1 & \dots & x1_n + x2_n \end{pmatrix} \cdot \begin{pmatrix} 1 & x1_1 + x2_1 \\ \vdots & \vdots \\ 1 & x1_n + x2_n \end{pmatrix} \cdot \begin{pmatrix} w_0 \\ w_1 \end{pmatrix}$$

■ 4

$$X^T Y = X^T X W$$
$$(X^T X)^{-1} (X^T X) W = (X^T X)^{-1} (X^T Y)$$
$$W = (X^T X)^{-1} (X^T Y)$$

Multivariate Regression

$$\mathbf{y} = \mathbf{w}_1 \mathbf{x}_1 + \mathbf{w}_2 \mathbf{x}_2 + \dots + \mathbf{w}_m \mathbf{x}_m$$

$$\mathbf{Y} = \mathbf{X} \mathbf{W}$$

$$\mathbf{W} = \vec{W}^T = \begin{pmatrix} w_1 \\ \vdots \\ w_m \end{pmatrix}; \text{ (% m \times 1 matrix \%)}$$

$$\vec{X} = (\mathbf{x}_1 \quad \dots \quad \mathbf{x}_m); \text{ (% m vector \%)}$$

$$\mathbf{X} = \begin{pmatrix} \vec{X}_1 \\ \vdots \\ \vec{X}_R \end{pmatrix} = \begin{pmatrix} \mathbf{x}_{1\,1} & \dots & \mathbf{x}_{1\,m} \\ \vdots & \ddots & \vdots \\ \mathbf{x}_{R\,1} & \dots & \mathbf{x}_{R\,m} \end{pmatrix}; \text{ (% R \times m matrix, R datapoints \%)}$$

$$\mathbf{Y} = \begin{pmatrix} y_1 \\ \vdots \\ y_R \end{pmatrix}; \text{ (% R \times 1 matrix \%)}$$

$$\mathbf{W} = (\mathbf{X}^T \mathbf{X})^{-1} (\mathbf{X}^T \mathbf{Y})$$

$$(\mathbf{X}^T \mathbf{X})_{ij} \Rightarrow \sum_{k=1}^R \mathbf{x}_{ki} \mathbf{x}_{kj}; \text{ (% m \times m matrix \%)}$$

$$(\mathbf{X}^T \mathbf{Y})_i \Rightarrow \sum_{k=1}^R \mathbf{x}_{ki} y_k; \text{ (% m - element vector \%)}$$

Sphere

$$W[[1]] \, x^2 + W[[1]] \, y^2 + W[[1]] \, z^2 - 2 \, W[[2]] \, x - 2 \, W[[3]] \, y - 2 \, W[[4]] \, z + W[[5]] == 1$$

$$W[[1]] \, (x^2 + y^2 + z^2) - 2 \, W[[2]] \, x - 2 \, W[[3]] \, y - 2 \, W[[4]] \, z + W[[5]] == 1$$

$$w1 \left(x - \frac{w2}{w1}\right)^2 + w1 \left(y - \frac{w3}{w1}\right)^2 + w1 \left(z - \frac{w4}{w1}\right)^2 + w5 - \frac{w2^2}{w1} - \frac{w3^2}{w1} - \frac{w4^2}{w1} == 1$$

$$(x - a)^2 + (y - b)^2 + (z - c)^2 == r^2$$

$$\frac{(x - a)^2}{r^2} + \frac{(y - b)^2}{r^2} + \frac{(z - c)^2}{r^2} == 1$$

$$\frac{x^2}{r^2} + \frac{y^2}{r^2} + \frac{z^2}{r^2} - \frac{2 \, a \, x}{r^2} - \frac{2 \, b \, y}{r^2} - \frac{2 \, c \, z}{r^2} + \frac{a^2}{r^2} + \frac{b^2}{r^2} + \frac{c^2}{r^2} == 1$$

$$W[[1]] == \frac{1}{r^2};$$

$$W[[2]] = \frac{a}{r^2};$$

$$W[[3]] = \frac{b}{r^2};$$

$$W[[4]] = \frac{c}{r^2};$$

$$W[[5]] = \frac{a^2}{r^2} + \frac{b^2}{r^2} + \frac{c^2}{r^2}$$

$$r^2 = \frac{1}{W[[1]]};$$

$$a = \frac{W[[2]]}{W[[1]]};$$

$$b = \frac{W[[3]]}{W[[1]]};$$

$$c = \frac{W[[4]]}{W[[1]]};$$

■ $W = (\mathbf{X}^T \mathbf{X})^{-1} (\mathbf{X}^T \mathbf{y})$

$$\mathbf{x} = [x^2 + y^2 + z^2, \, -2 \, x, \, -2 \, y, \, -2 \, z, \, 1]$$

$$\mathbf{y} = [1]$$

$$\mathbf{W} = \begin{pmatrix} \mathbf{AA} \\ \mathbf{BB} \\ \mathbf{CC} \\ \mathbf{DD} \\ \mathbf{EE} \end{pmatrix}$$

$$\mathbf{X} = \begin{pmatrix} \mathbf{A1} & \mathbf{B1} & \mathbf{C1} & \mathbf{D1} & \mathbf{E1} \\ \mathbf{At} & \mathbf{Bt} & \mathbf{Ct} & \mathbf{Dt} & \mathbf{Et} \end{pmatrix}; \mathbf{Y} = \begin{pmatrix} 1 \\ 1 \end{pmatrix};$$

$$\mathbf{W} = (\mathbf{X}^T.\mathbf{X})^{-1} . (\mathbf{X}^T.\mathbf{Y});$$

■ 1

```
In[55]:= X = ( A0 B0 C0 D0 1 ); Y = (1);  
W = (X^T.X)^-1. (X^T.Y);  
W // MatrixForm
```

```
Out[59]//MatrixForm=  

$$\begin{pmatrix} \frac{5}{A0} \\ \frac{5}{B0} \\ \frac{5}{C0} \\ \frac{5}{D0} \\ 5 \end{pmatrix}$$

```

■ 2

```
In[52]:= X = ( A0 B0 C0 D0 1  
A1 B1 C1 D1 1 ); Y = ( 1  
1 );  
W = (X^T.X)^-1. (X^T.Y);  
W // MatrixForm
```

```
Out[54]//MatrixForm=  

$$\begin{pmatrix} \frac{2}{A0+A1} + \frac{A0+A1}{A0^2+A1^2} + \frac{B0+B1}{A0 B0+A1 B1} + \frac{C0+C1}{A0 C0+A1 C1} + \frac{D0+D1}{A0 D0+A1 D1} \\ \frac{2}{B0+B1} + \frac{A0+A1}{A0 B0+A1 B1} + \frac{B0+B1}{B0^2+B1^2} + \frac{C0+C1}{B0 C0+B1 C1} + \frac{D0+D1}{B0 D0+B1 D1} \\ \frac{2}{C0+C1} + \frac{A0+A1}{A0 C0+A1 C1} + \frac{B0+B1}{B0 C0+B1 C1} + \frac{C0+C1}{C0^2+C1^2} + \frac{D0+D1}{C0 D0+C1 D1} \\ \frac{2}{D0+D1} + \frac{A0+A1}{A0 D0+A1 D1} + \frac{B0+B1}{B0 D0+B1 D1} + \frac{C0+C1}{C0 D0+C1 D1} + \frac{D0+D1}{D0^2+D1^2} \\ 5 \end{pmatrix}$$

```

■ 3

```
In[49]:= X = ( A0 B0 C0 D0 1  
A1 B1 C1 D1 1  
A2 B2 C2 D2 1 ); Y = ( 1  
1  
1 );  
W = (X^T.X)^-1. (X^T.Y);  
W // MatrixForm
```

```
Out[51]//MatrixForm=  

$$\begin{pmatrix} \frac{3}{A0+A1+A2} + \frac{A0+A1+A2}{A0^2+A1^2+A2^2} + \frac{B0+B1+B2}{A0 B0+A1 B1+A2 B2} + \frac{C0+C1+C2}{A0 C0+A1 C1+A2 C2} + \frac{D0+D1+D2}{A0 D0+A1 D1+A2 D2} \\ \frac{3}{B0+B1+B2} + \frac{A0+A1+A2}{A0 B0+A1 B1+A2 B2} + \frac{B0+B1+B2}{B0^2+B1^2+B2^2} + \frac{C0+C1+C2}{B0 C0+B1 C1+B2 C2} + \frac{D0+D1+D2}{B0 D0+B1 D1+B2 D2} \\ \frac{3}{C0+C1+C2} + \frac{A0+A1+A2}{A0 C0+A1 C1+A2 C2} + \frac{B0+B1+B2}{B0 C0+B1 C1+B2 C2} + \frac{C0+C1+C2}{C0^2+C1^2+C2^2} + \frac{D0+D1+D2}{C0 D0+C1 D1+C2 D2} \\ \frac{3}{D0+D1+D2} + \frac{A0+A1+A2}{A0 D0+A1 D1+A2 D2} + \frac{B0+B1+B2}{B0 D0+B1 D1+B2 D2} + \frac{C0+C1+C2}{C0 D0+C1 D1+C2 D2} + \frac{D0+D1+D2}{D0^2+D1^2+D2^2} \\ 5 \end{pmatrix}$$

```

■ 4

```
In[46]:= X = ( A0 B0 C0 D0 1  
A1 B1 C1 D1 1  
A2 B2 C2 D2 1  
A3 B3 C3 D3 1 ); Y = ( 1  
1  
1  
1 );  
W = (X^T.X)^-1. (X^T.Y);  
W // MatrixForm
```

```
Out[48]//MatrixForm=  

$$\begin{pmatrix} \frac{4}{A0+A1+A2+A3} + \frac{A0+A1+A2+A3}{A0^2+A1^2+A2^2+A3^2} + \frac{B0+B1+B2+B3}{A0 B0+A1 B1+A2 B2+A3 B3} + \frac{C0+C1+C2+C3}{A0 C0+A1 C1+A2 C2+A3 C3} + \frac{D0+D1+D2+D3}{A0 D0+A1 D1+A2 D2+A3 D3} \\ \frac{4}{B0+B1+B2+B3} + \frac{A0+A1+A2+A3}{A0 B0+A1 B1+A2 B2+A3 B3} + \frac{B0+B1+B2+B3}{B0^2+B1^2+B2^2+B3^2} + \frac{C0+C1+C2+C3}{B0 C0+B1 C1+B2 C2+B3 C3} + \frac{D0+D1+D2+D3}{B0 D0+B1 D1+B2 D2+B3 D3} \\ \frac{4}{C0+C1+C2+C3} + \frac{A0+A1+A2+A3}{A0 C0+A1 C1+A2 C2+A3 C3} + \frac{B0+B1+B2+B3}{B0 C0+B1 C1+B2 C2+B3 C3} + \frac{C0+C1+C2+C3}{C0^2+C1^2+C2^2+C3^2} + \frac{D0+D1+D2+D3}{C0 D0+C1 D1+C2 D2+C3 D3} \\ \frac{4}{D0+D1+D2+D3} + \frac{A0+A1+A2+A3}{A0 D0+A1 D1+A2 D2+A3 D3} + \frac{B0+B1+B2+B3}{B0 D0+B1 D1+B2 D2+B3 D3} + \frac{C0+C1+C2+C3}{C0 D0+C1 D1+C2 D2+C3 D3} + \frac{D0+D1+D2+D3}{D0^2+D1^2+D2^2+D3^2} \\ 5 \end{pmatrix}$$

```

■ t

```
At = x^2 + y^2 + z^2;  
Bt = -2 x;  
Ct = -2 y;  
Dt = -2 z;  
Et = 1;
```

$$X = \begin{pmatrix} A1 & B1 & C1 & D1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ At & Bt & Ct & Dt & 1 \end{pmatrix}; Y = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix};$$

```
W = (X^T.X)^-1. (X^T.Y);  
W // MatrixForm
```

$$W[[1]] = \frac{t}{A1 + \dots + At} + \frac{A1 + \dots + At}{A1^2 + \dots + At^2} + \frac{B1 + \dots + Bt}{A1 B1 + \dots + At Bt} + \frac{C1 + \dots + Ct}{A1 C1 + \dots + At Ct} + \frac{D1 + \dots + Dt}{A1 D1 + \dots + At Dt}$$

$$W[[2]] = \frac{t}{B1 + \dots + Bt} + \frac{A1 + \dots + At}{A1 B1 + \dots + At Bt} + \frac{B1 + \dots + Bt}{B1^2 + \dots + Bt^2} + \frac{C1 + \dots + Ct}{B1 C1 + \dots + Bt Ct} + \frac{D1 + \dots + Dt}{B1 D1 + \dots + Bt Dt}$$
$$W[[3]] = \frac{t}{C1 + \dots + Ct} + \frac{A1 + \dots + At}{A1 C1 + \dots + At Ct} + \frac{B1 + \dots + Bt}{B1 C1 + \dots + Bt Ct} + \frac{C1 + \dots + Ct}{C1^2 + \dots + Ct^2} + \frac{D1 + \dots + Dt}{C1 D1 + \dots + Ct Dt}$$
$$W[[4]] = \frac{t}{D1 + \dots + Dt} + \frac{A1 + \dots + At}{A1 D1 + \dots + At Dt} + \frac{B1 + \dots + Bt}{B1 D1 + \dots + Bt Dt} + \frac{C1 + \dots + Ct}{C1 D1 + \dots + Ct Dt} + \frac{D1 + \dots + Dt}{D1^2 + \dots + Dt^2}$$
$$W[[5]] = 5$$
$$r^2 = \frac{1}{W[[1]]};$$
$$a = \frac{W[[2]]}{W[[1]]};$$
$$b = \frac{W[[3]]}{W[[1]]};$$
$$c = \frac{W[[4]]}{W[[1]]};$$

■ $W = (x^T x)^{-1} (x^T y)$

$$x = [x^2 + y^2 + z^2, -2 x, -2 y, -2 z, 1]$$
$$y = [1]$$
$$W[[1]] = \frac{t}{A1 + \dots + At} + \frac{A1 + \dots + At}{A1^2 + \dots + At^2} + \frac{B1 + \dots + Bt}{A1 B1 + \dots + At Bt} + \frac{C1 + \dots + Ct}{A1 C1 + \dots + At Ct} + \frac{D1 + \dots + Dt}{A1 D1 + \dots + At Dt}$$
$$W[[2]] = \frac{t}{B1 + \dots + Bt} + \frac{A1 + \dots + At}{A1 B1 + \dots + At Bt} + \frac{B1 + \dots + Bt}{B1^2 + \dots + Bt^2} + \frac{C1 + \dots + Ct}{B1 C1 + \dots + Bt Ct} + \frac{D1 + \dots + Dt}{B1 D1 + \dots + Bt Dt}$$
$$W[[3]] = \frac{t}{C1 + \dots + Ct} + \frac{A1 + \dots + At}{A1 C1 + \dots + At Ct} + \frac{B1 + \dots + Bt}{B1 C1 + \dots + Bt Ct} + \frac{C1 + \dots + Ct}{C1^2 + \dots + Ct^2} + \frac{D1 + \dots + Dt}{C1 D1 + \dots + Ct Dt}$$
$$W[[4]] = \frac{t}{D1 + \dots + Dt} + \frac{A1 + \dots + At}{A1 D1 + \dots + At Dt} + \frac{B1 + \dots + Bt}{B1 D1 + \dots + Bt Dt} + \frac{C1 + \dots + Ct}{C1 D1 + \dots + Ct Dt} + \frac{D1 + \dots + Dt}{D1^2 + \dots + Dt^2}$$
$$W[[5]] = 5$$
$$r^2 = \frac{1}{W[[1]]};$$
$$a = \frac{W[[2]]}{W[[1]]};$$
$$b = \frac{W[[3]]}{W[[1]]};$$
$$c = \frac{W[[4]]}{W[[1]]};$$

Elliptical Sphere

$$AA\,x^2 + BB\,y^2 + CC\,z^2 + 2\,DD\,x + 2\,EE\,y + 2\,FF\,z = 1$$
$$AA\,(x - a)^2 + BB\,(y - b)^2 + CC\,(z - c)^2 = 1$$
$$AA\,x^2 + BB\,y^2 + CC\,z^2 - 2\,a\,AA\,x - 2\,b\,BB\,y - 2\,c\,CC\,z + a^2\,AA + b^2\,BB + c^2\,CC = 1$$
$$\text{Inverse}\Bigg[\begin{pmatrix} AA & 0 & 0 \\ 0 & BB & 0 \\ 0 & 0 & CC \end{pmatrix}\Bigg] \cdot \begin{pmatrix} DD \\ EE \\ FF \end{pmatrix} // \text{MatrixForm}$$
$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} \frac{DD}{AA} \\ \frac{EE}{BB} \\ \frac{FF}{CC} \end{pmatrix}$$

■ $W = (X^T X)^{-1} (X^T Y)$

$$x = [x^2, y^2, z^2, 2 x, 2 y, 2 z]$$
$$W = \begin{pmatrix} AA \\ BB \\ CC \\ DD \\ EE \\ FF \end{pmatrix}$$
$$a = \frac{AA}{DD} = W[[4]] / W[[1]];$$
$$b = \frac{BB}{EE} = W[[5]] / W[[2]];$$
$$c = \frac{CC}{FF} = W[[6]] / W[[3]];$$
$$X = \begin{pmatrix} A_o & B_o & C_o & D_o & E_o & F_o \\ A_t & B_t & C_t & D_t & E_t & F_t \end{pmatrix}; Y = \begin{pmatrix} 1 \\ 1 \end{pmatrix};$$
$$W = (X^T.X)^{-1} . (X^T.Y);$$

■ 0

```
In[54]:= X = ( A0 B0 C0 D0 E0 F0 ); Y = (1);
W = (Xᵀ.X)⁻¹.(Xᵀ.Y);
W // MatrixForm
```

```
Out[56]//MatrixForm=
{{ 6
A0 }, { 6
B0 }, { 6
C0 }, { 6
D0 }, { 6
E0 }, { 6
F0 }}.1
```

■ 1

```
In[57]:= X = ( A0 B0 C0 D0 E0 F0
A1 B1 C1 D1 E1 F1 ); Y = ( 1
1 );
W = (Xᵀ.X)⁻¹.(Xᵀ.Y);
W // MatrixForm
```

```
Out[59]//MatrixForm=
(
A0+A1
A0²+A1² + B0+B1
A0 B0+A1 B1 + C0+C1
A0 C0+A1 C1 + D0+D1
A0 D0+A1 D1 + E0+E1
A0 E0+A1 E1 + F0+F1
A0 B0+A1 B1 + B0²+B1² + C0+C1
A0 C0+B1 C1 + B0 D0+B1 D1 + E0+E1
A0 E0+B1 E1 + F0+F1
A0 D0+A1 D1 + B0 C0+B1 C1 + C0²+C1² + D0+D1
A0 D0+B1 D1 + C0 E0+C1 E1 + F0+F1
A0 E0+A1 E1 + B0 D0+B1 D1 + C0 D0+C1 D1 + D0²+D1² + E0+E1
A0 E0+B1 E1 + C0 E0+C1 E1 + D0 E0+D1 E1 + E0²+E1² + F0+F1
A0 F0+A1 F1 + B0 F0+B1 F1 + C0 F0+C1 F1 + D0 F0+D1 F1 + E0 F0+E1 F1 + F0²+F1²
)
```

■ 2

```
In[60]:= X = ( A0 B0 C0 D0 E0 F0
A1 B1 C1 D1 E1 F1
A2 B2 C2 D2 E2 F2 ); Y = ( 1
1
1 );
W = (Xᵀ.X)⁻¹.(Xᵀ.Y);
W // MatrixForm
```

```
Out[62]//MatrixForm=
(
A0+A1+A2
A0²+A1²+A2² + B0+B1+B2
A0 B0+A1 B1+A2 B2 + C0+C1+C2
A0 C0+A1 C1+A2 C2 + D0+D1+D2
A0 D0+A1 D1+A2 D2 + E0+E1+E2
A0 E0+A1 E1+A2 E2 + F0+F1+F2
A0 B0+A1 B1+A2 B2 + B0²+B1²+B2² + C0+C1+C2
A0 C0+B1 C1+B2 C2 + C0²+C1²+C2² + D0+D1+D2
A0 D0+B1 D1+B2 D2 + C0 D0+C1 D1+C2 D2 + E0+E1+E2
A0 E0+B1 E1+B2 E2 + C0 E0+C1 E1+C2 E2 + F0+F1+F2
A0 F0+A1 F1+A2 F2 + B0 F0+B1 F1+B2 F2 + C0 F0+C1 F1+C2 F2 + D0 F0+D1 F1+D2 F2 + E0 F0+E1 F1+E2 F2 + F0²+F1²+F2²
)
```

■ t

```
In[69]:= X = ( A0 B0 C0 D0 E0 F0
A1 B1 C1 D1 E1 F1
At Bt Ct Dt Et Ft ); Y = ( 1
1
1 );
W = (Xᵀ.X)⁻¹.(Xᵀ.Y);
W // MatrixForm
```

```
Out[71]//MatrixForm=
(
A0+A1+At
A0²+A1²+At² + B0+B1+Bt
A0 B0+A1 B1+At Bt + C0+C1+Ct
A0 C0+A1 C1+At Ct + D0+D1+Dt
A0 D0+A1 D1+At Dt + E0+E1+Et
A0 E0+A1 E1+At Et + F0+F1+Ft
A0 B0+A1 B1+At Bt + B0²+B1²+Bt² + C0+C1+Ct
A0 C0+B1 C1+Bt Ct + C0²+C1²+Ct² + D0+D1+Dt
A0 D0+B1 D1+Bt Dt + C0 D0+C1 D1+Ct Ct + E0+E1+Et
A0 E0+B1 E1+Bt Et + C0 E0+C1 E1+Ct Ct + F0+F1+Ft
A0 F0+A1 F1+At Ft + B0 F0+B1 F1+Bt Ft + C0 F0+C1 F1+Ct Ft + D0 F0+D1 F1+Dt Ft + E0 F0+E1 F1+Et Ft + F0²+F1²+Ft²
)
```

W[[1]] = $\frac{A0 + \dots + At}{A0^2 + \dots + At^2} + \frac{B0 + \dots + Bt}{A0 B0 + \dots + At Bt} + \frac{C0 + \dots + Ct}{A0 C0 + \dots + At Ct} + \frac{D0 + \dots + Dt}{A0 D0 + \dots + At Dt} + \frac{E0 + \dots + Et}{A0 E0 + \dots + At Et} + \frac{F0 + \dots + Ft}{A0 F0 + \dots + At Ft}$

W[[2]] = $\frac{A0 + \dots + At}{A0 B0 + \dots + At Bt} + \frac{B0 + \dots + Bt}{B0^2 + \dots + Bt^2} + \frac{C0 + \dots + Ct}{B0 C0 + \dots + Bt Ct} + \frac{D0 + \dots + Dt}{B0 D0 + \dots + Bt Dt} + \frac{E0 + \dots + Et}{B0 E0 + \dots + Bt Et} + \frac{F0 + \dots + Ft}{B0 F0 + \dots + Bt Ft}$

W[[3]] = $\frac{A0 + \dots + At}{A0 C0 + \dots + At Ct} + \frac{B0 + \dots + Bt}{B0 C0 + \dots + Bt Ct} + \frac{C0 + \dots + Ct}{C0^2 + \dots + Ct^2} + \frac{D0 + \dots + Dt}{C0 D0 + \dots + Ct Dt} + \frac{E0 + \dots + Et}{C0 E0 + \dots + Ct Et} + \frac{F0 + \dots + Ft}{C0 F0 + \dots + Ct Ft}$

W[[4]] = $\frac{A0 + \dots + At}{A0 D0 + \dots + At Dt} + \frac{B0 + \dots + Bt}{B0 D0 + \dots + Bt Dt} + \frac{C0 + \dots + Ct}{C0 D0 + \dots + Ct Dt} + \frac{D0 + \dots + Dt}{D0^2 + \dots + Dt^2} + \frac{E0 + \dots + Et}{D0 E0 + \dots + Dt Et} + \frac{F0 + \dots + Ft}{D0 F0 + \dots + Dt Ft}$

W[[5]] = $\frac{A0 + \dots + At}{A0 E0 + \dots + At Et} + \frac{B0 + \dots + Bt}{B0 E0 + \dots + Bt Et} + \frac{C0 + \dots + Ct}{C0 E0 + \dots + Ct Et} + \frac{D0 + \dots + Dt}{D0 E0 + \dots + Dt Et} + \frac{E0 + \dots + Et}{E0^2 + \dots + Et^2} + \frac{F0 + \dots + Ft}{E0 F0 + \dots + Et Ft}$

W[[6]] = $\frac{A0 + \dots + At}{A0 F0 + \dots + At Ft} + \frac{B0 + \dots + Bt}{B0 F0 + \dots + Bt Ft} + \frac{C0 + \dots + Ct}{C0 F0 + \dots + Ct Ft} + \frac{D0 + \dots + Dt}{D0 F0 + \dots + Dt Ft} + \frac{E0 + \dots + Et}{E0 F0 + \dots + Et Ft} + \frac{F0 + \dots + Ft}{F0^2 + \dots + Ft^2}$