

Mathematical Tables

■ Trigonometric Identities

$$\sin^2[\alpha] + \cos^2[\alpha] = 1$$

$$\sin[\alpha + \beta] = \sin[\alpha] \cos[\beta] + \cos[\alpha] \sin[\beta]$$

$$\sin[\alpha - \beta] = \sin[\alpha] \cos[\beta] - \cos[\alpha] \sin[\beta]$$

$$\cos[\alpha + \beta] = \cos[\alpha] \cos[\beta] - \sin[\alpha] \sin[\beta]$$

$$\cos[\alpha - \beta] = \cos[\alpha] \cos[\beta] + \sin[\alpha] \sin[\beta]$$

$$\sin[2\alpha] = 2 \sin[\alpha] \cos[\alpha]$$

$$\cos[2\alpha] = \cos^2[\alpha] - \sin^2[\alpha] = 2 \cos^2[\alpha] - 1 = 1 - 2 \sin^2[\alpha]$$

$$\sin[\alpha] \sin[\beta] = \frac{1}{2} \cos[\alpha - \beta] - \frac{1}{2} \cos[\alpha + \beta]$$

$$\cos[\alpha] \cos[\beta] = \frac{1}{2} \cos[\alpha - \beta] + \frac{1}{2} \cos[\alpha + \beta]$$

$$\sin[\alpha] \cos[\beta] = \frac{1}{2} \sin[\alpha + \beta] + \frac{1}{2} \sin[\alpha - \beta]$$

$$\cos[\alpha] \sin[\beta] = \frac{1}{2} \sin[\alpha + \beta] - \frac{1}{2} \sin[\alpha - \beta]$$

$$\sin^2[\alpha] = \frac{1}{2} (1 - \cos[2\alpha])$$

$$\cos^2[\alpha] = \frac{1}{2} (1 + \cos[2\alpha])$$

$$e^{\pm i\alpha} = \cos[\alpha] \pm i \sin[\alpha]$$

$$\cos[\alpha] = \frac{(e^{i\alpha} + e^{-i\alpha})}{2}$$

$$\sin[\alpha] = \frac{(e^{i\alpha} - e^{-i\alpha})}{2i}$$

$$\sin[\alpha] = \cos\left[\alpha - \frac{\pi}{2}\right]$$

■ Indefinite Integrals

$$\int u \, dv = uv - \int v \, du \quad \text{where } u \text{ and } v \text{ are functions of } x$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1}$$

$$\int x^{-1} \, dx = \ln|x|$$

$$\int e^{ax} dx = \frac{e^{ax}}{a}$$

$$\int \ln[x] dx = x \ln[x] - x$$

$$\int (a^2 + x^2)^{-1} dx = \frac{1}{a} \tan\left[\frac{x}{a}\right]^{-1}$$

$$\int \frac{(\ln[x])^n}{x} dx = \frac{\ln[x]^{n+1}}{n+1}$$

$$\int x^n \ln[ax] dx = \frac{x^{n+1}}{n+1} \ln[ax] - \frac{x^{n+1}}{(n+1)^2}$$

$$\int x e^{ax} dx = e^{ax} \frac{(ax - 1)}{a^2}$$

$$\int x^2 e^{ax} dx = e^{ax} \frac{(a^2 x^2 - 2ax + 2)}{a^3}$$

$$\int \sin[ax] dx = -\frac{1}{a} \cos[ax]$$

$$\int \cos[ax] dx = \frac{1}{a} \sin[ax]$$

$$\int \sin[ax]^2 dx = \frac{x}{2} - \frac{\sin[2ax]}{4a}$$

$$\int x \sin[ax] dx = \frac{1}{a^2} (\sin[ax] - ax \cos[ax])$$

$$\int x^2 \sin[ax] dx = \frac{1}{a^3} (2ax \sin[ax] + 2 \cos[ax] - a^2 x^2 \sin[ax])$$

$$\int \cos[ax]^2 dx = \frac{x}{2} + \frac{\sin[2ax]}{4a}$$

$$\int x \cos[ax] dx = \frac{1}{a^2} (\cos[ax] + ax \sin[ax])$$

$$\int x^2 \cos[ax] dx = \frac{1}{a^3} (2ax \cos[ax] - 2 \sin[ax] + a^2 x^2 \sin[ax])$$

■ Definite Integrals

$$\int_0^\infty t^{n-1} e^{-(a+1)t} dt = \frac{\Gamma[n]}{(a+1)^n} \quad n > 0, a > -1$$

$$\Gamma[n] = (n-1)! \quad \text{if } n \text{ is an integer, } n > 0$$

$$\Gamma\left[\frac{1}{2}\right] = \sqrt{\pi}$$

$$\Gamma\left[n + \frac{1}{2}\right] = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^n} \sqrt{\pi} \quad n = 1, 2, 3, \dots$$

$$\int_0^\infty e^{-\alpha^2 x^2} dx = \frac{\sqrt{\pi}}{2\alpha}$$

$$\int_0^{\infty} x e^{-\alpha^2 x^2} dx = \frac{1}{2 \alpha^2}$$

$$\int_0^{\infty} x^2 e^{-\alpha^2 x^2} dx = \frac{\sqrt{\pi}}{4 \alpha^3}$$

$$\int_0^{\infty} x^n e^{-\alpha^2 x^2} dx = \frac{\Gamma[\frac{n+1}{2}]}{2 \alpha^{n+1}}$$

$$\int_0^{\infty} \frac{a}{(a^2 + x^2)} dx = \frac{\pi}{2} \quad \text{if } a > 0$$

$$\int_0^{\infty} \frac{\sin[ax]^2}{x^2} dx = |a| \frac{\pi}{2} \quad \text{if } a > 0$$