

Multivariate Normal Distribution

$$\frac{1}{\sqrt{(2\pi)^k |\Sigma|}} e^{-\frac{1}{2} (X-\mu)^\top \Sigma^{-1} (X-\mu)}$$

$$X = \begin{pmatrix} x \\ y \end{pmatrix};$$

$$M = \begin{pmatrix} \mu_x \\ \mu_y \end{pmatrix};$$

$$\Sigma = \begin{pmatrix} \sigma_x^2 & \rho \sigma_x \sigma_y \\ \rho \sigma_x \sigma_y & \sigma_y^2 \end{pmatrix};$$

$$\Sigma = \begin{pmatrix} Sx^2 & r Sx Sy \\ r Sx Sy & Sy^2 \end{pmatrix};$$

Det[Σ] // Simplify

$$-(-1 + r^2) Sx^2 Sy^2$$

Inverse[Σ] // MatrixForm

$$\begin{pmatrix} \frac{Sy^2}{Sx^2 Sy^2 - r^2 Sx^2 Sy^2} & -\frac{r Sx Sy}{Sx^2 Sy^2 - r^2 Sx^2 Sy^2} \\ -\frac{r Sx Sy}{Sx^2 Sy^2 - r^2 Sx^2 Sy^2} & \frac{Sx^2}{Sx^2 Sy^2 - r^2 Sx^2 Sy^2} \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{Sx^2 (1-r^2)} & -\frac{r}{Sx Sy (1-r^2)} \\ -\frac{r}{Sx Sy (1-r^2)} & \frac{1}{Sy^2 (1-r^2)} \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{Sx^2 (1-r^2)} & -\frac{r}{Sx Sy (1-r^2)} \\ -\frac{r}{Sx Sy (1-r^2)} & \frac{1}{Sy^2 (1-r^2)} \end{pmatrix}$$

(X - M) // MatrixForm

$$\begin{pmatrix} X - \mu_x \\ Y - \mu_y \end{pmatrix}$$

$$-\frac{1}{2} (X - M)^\top \cdot \Sigma^{-1} \cdot (X - M)$$

$$XM = \begin{pmatrix} XmUx \\ XmUy \end{pmatrix};$$

XM^T.Inverse[Σ].XM // Simplify // MatrixForm

$$\left(-\frac{Sy^2 XmUx^2 - 2 r Sx Sy XmUx XmUy + Sx^2 XmUy^2}{(-1+r^2) Sx^2 Sy^2} \right)$$

$$\left\{ \left\{ -\frac{XmUx^2}{(-1+r^2) Sx^2} + \frac{2 r XmUx XmUy}{(-1+r^2) Sx Sy} - \frac{XmUy^2}{(-1+r^2) Sy^2} \right\} \right\}$$

$$\frac{1}{\sqrt{(2\pi)^2 \text{Det}[\Sigma]}} e^{-\frac{1}{2} (X-M)^\top \cdot \text{Inverse}[\Sigma] \cdot (X-M)}$$

$$\frac{1}{2\pi \sigma_x \sigma_y \sqrt{1-\rho^2}} e^{-\frac{1}{2(1-\rho^2)} \left(\frac{(x-\mu_x)^2}{\sigma_x^2} - \frac{2\rho(x-\mu_x)(y-\mu_y)}{\sigma_x \sigma_y} + \frac{(y-\mu_y)^2}{\sigma_y^2} \right)}$$

■ Determinate

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A =  $\begin{pmatrix} \sigma_x^2 & \rho \sigma_x \sigma_y \\ \rho \sigma_x \sigma_y & \sigma_y^2 \end{pmatrix};$ 
Det[A] // Simplify
A-1 ⇒ Inverse[A] // MatrixForm

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Bivariate Normal Distribution

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X = {x1, x2, x3, x4, x5};
Y = {y1, y2, y3, y4, y5};
n = Length[X];

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$$\mu_x = \sum_{i=1}^n \frac{\text{Part}[X, i]}{n};$$

$$\mu_y = \sum_{i=1}^n \frac{\text{Part}[Y, i]}{n};$$

$$\Sigma = \begin{pmatrix} S_{XX} & \rho \sigma_x \sigma_y \\ \rho \sigma_x \sigma_y & \sigma_y^2 \end{pmatrix};$$

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S_XX = Part[{X - MuX}.{X - MuX}T, 1, 1] / (n - 1);
S_X = Sqrt[S_XX];

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S_XY = Part[{X - MuX}.{Y - MuY}T, 1, 1] / (n - 1);

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S_YY = Part[{Y - MuY}.{Y - MuY}T, 1, 1] / (n - 1);

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S_Y = Sqrt[S_YY];

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$$R = \frac{S_{XY}}{S_X S_Y};$$

$$\frac{1}{2 \pi S_{XX} S_{YY} \sqrt{1 - R^2}} e^{-\frac{1}{2(1-R^2)} \left(\left(\frac{x-MX}{S_{XX}} \right)^2 - \frac{2R(x-MX)(y-MY)}{S_{XX} S_{YY}} + \left(\frac{y-MY}{S_{YY}} \right)^2 \right)} // \text{InputForm}$$

Bivariate Normal Distribution

$$X_n = \begin{pmatrix} x_n \\ y_n \end{pmatrix}$$

$$\mu_k = \begin{pmatrix} \mu_x \\ \mu_y \end{pmatrix}$$

$$\Sigma_k = \begin{pmatrix} \sigma_x^2 & \rho \sigma_x \sigma_y \\ \rho \sigma_x \sigma_y & \sigma_y^2 \end{pmatrix}$$

$$\sigma_x^2 = E[(X - \mu_x)^2]$$

$$\sigma_y^2 = E[(Y - \mu_y)^2]$$

$$\rho = \frac{\mathbf{E}[(X - \mu_x)(Y - \mu_y)]}{\sigma_x \sigma_y}$$

$$\mathbf{E}[X] = \frac{1}{n-1} \sum_{i=0}^n X_i$$

$$\mathbf{Det}[\Sigma] = \sigma_x^2 \sigma_y^2 (1 - \rho^2)$$

$$\Sigma^{-1} = \frac{1}{(1 - \rho^2)} \begin{pmatrix} \frac{1}{\sigma_x^2} & \frac{-1}{\sigma_x \sigma_y} \\ \frac{-1}{\sigma_x \sigma_y} & \frac{1}{\sigma_y^2} \end{pmatrix}$$

$$\mathbf{N}[\mu_j, \Sigma_j] = \frac{1}{2 \pi \sigma_x \sigma_y \sqrt{1 - \rho^2}} \mathbf{e}^{-\frac{1}{2(1 - \rho^2)} \left(\frac{(X - \mu_x)^2}{\sigma_x^2} + \frac{(Y - \mu_y)^2}{\sigma_y^2} - \frac{2 \rho (X - \mu_x)(Y - \mu_y)}{\sigma_x \sigma_y} \right)}$$