

3 Dimension Space

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3 D Equation

■ Point

$$P = \{x_p, y_p, z_p, 1\}$$

$$Q = \{x_q, y_q, z_q, 1\}$$

■ Line Equation

$$\frac{x - x_p}{x_q - x_p} = \frac{y - y_p}{y_q - y_p} = \frac{z - z_p}{z_q - z_p}$$

■ Plane(Normal Vector)

$$N = \{a, b, c, d\}$$

■ Line Equation

$$\frac{x - x_p}{a} = \frac{y - y_p}{b} = \frac{z - z_p}{c}$$

■ Plane Equation

$$N = \{a, b, c, d\}$$

$$X - P_0 = \{x - x_0, y - y_0, z - z_0, 1 - 1\}$$

$$N \cdot (X - P_0) = 0$$

$$\{a, b, c, d\} \cdot \{x - x_0, y - y_0, z - z_0, 0\} = 0$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$$ax + by + cz - ax_0 - by_0 - cz_0 = 0$$

$$ax + by + cz + d = 0$$

■ 3 Point → Plane (Normal Vector)

$$\mathbf{N} = \{a, b, c, d\}$$

$$\mathbf{P}_0 = \{x_0, y_0, z_0, 1\}$$

$$\mathbf{P}_1 = \{x_1, y_1, z_1, 1\}$$

$$\mathbf{P}_2 = \{x_2, y_2, z_2, 1\}$$

$$\mathbf{N} \cdot \mathbf{P}_0 = 0$$

$$\{a, b, c, d\} \cdot \{x_0, y_0, z_0, 1\} = 0$$

$$a x_0 + b y_0 + c z_0 + d = 0$$

$$\mathbf{N} \cdot \mathbf{P}_1 = 0$$

$$\{a, b, c, d\} \cdot \{x_1, y_1, z_1, 1\} = 0$$

$$a x_1 + b y_1 + c z_1 + d = 0$$

$$\mathbf{N} \cdot \mathbf{P}_2 = 0$$

$$\{a, b, c, d\} \cdot \{x_2, y_2, z_2, 1\} = 0$$

$$a x_2 + b y_2 + c z_2 + d = 0$$

$$\mathbf{N} = \mathbf{P}_0 \wedge \mathbf{P}_1 \wedge \mathbf{P}_2$$

$$\mathbf{N} = \text{Det} \begin{bmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3 & \mathbf{e}_4 \\ x_0 & y_0 & z_0 & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \end{bmatrix}$$

$$\mathbf{e}_1 = \{1, 0, 0, 0\}$$

$$\mathbf{e}_2 = \{0, 1, 0, 0\}$$

$$\mathbf{e}_3 = \{0, 0, 1, 0\}$$

$$\mathbf{e}_4 = \{0, 0, 0, 1\}$$

■ 3 Plane (Normal Vector) → Point

$$\mathbf{P} = \{x, y, z, 1\}$$

$$\mathbf{N}_0 = \{a_0, b_0, c_0, d_0\}$$

$$\mathbf{N}_1 = \{a_1, b_1, c_1, d_1\}$$

$$\mathbf{N}_2 = \{a_2, b_2, c_2, d_2\}$$

$$\mathbf{P} = \mathbf{N}_0 \wedge \mathbf{N}_1 \wedge \mathbf{N}_2$$

$$\mathbf{P} = \text{Det} \begin{bmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3 & \mathbf{e}_4 \\ a_0 & b_0 & c_0 & d_0 \\ a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \end{bmatrix}$$

$$\mathbf{e}_1 = \{1, 0, 0, 0\}$$

$$\mathbf{e}_2 = \{0, 1, 0, 0\}$$

$$\mathbf{e}_3 = \{0, 0, 1, 0\}$$

$$\mathbf{e}_4 = \{0, 0, 0, 1\}$$

■ 2 Line→ Plane (Normal Vector)

$$N = \{a, b, c, d\}$$

$$L_1 = \{l_1, m_1, n_1\}, \quad \frac{x - x_1}{l_1} = \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1} = t_1$$

$$\{x_1 + l_1 t_1, y_1 + m_1 t_1, z_1 + n_1 t_1, 1\}$$

$$L_2 = \{l_2, m_2, n_2\}, \quad \frac{x - x_2}{l_2} = \frac{y - y_2}{m_2} = \frac{z - z_2}{n_2} = t_2$$

$$\{x_2 + l_2 t_2, y_2 + m_2 t_2, z_2 + n_2 t_2, 1\}$$

$$N \cdot L_1 = 0$$

$$N \cdot L_2 = 0$$

$$a x + b y + c z + d = 0$$

■ 2 Line Angle

$$\cos[\theta] = \frac{L_1 \cdot L_2}{\text{Norm}[L_1] \text{Norm}[L_2]}$$

Equation Solution

Solve[

$$\{a_0 x + b_0 y + c_0 z + d_0 = 0, a_1 x + b_1 y + c_1 z + d_1 = 0, a_2 x + b_2 y + c_2 z + d_2 = 0\}, \{x, y, z\};$$

$$\{a_3, b_3, c_3, d_3\} = \{a_0, b_0, c_0, d_0\} \times \{a_1, b_1, c_1, d_1\} \times \{a_2, b_2, c_2, d_2\};$$

$$\{a_3, b_3, c_3, d_3\} = \text{Det} \left[\begin{pmatrix} e_1 & e_2 & e_3 & e_4 \\ a_0 & b_0 & c_0 & d_0 \\ a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \end{pmatrix} \right]$$

$$\left\{ \frac{a_3}{d_3}, \frac{b_3}{d_3}, \frac{c_3}{d_3}, 1 \right\}$$

Homogeneous Coordinates Matrix

■ Reduction, Extension, Movement

$$\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha & 0 & 0 & t_x \\ 0 & \beta & 0 & t_y \\ 0 & 0 & \gamma & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha x + t_x \\ \beta y + t_y \\ \gamma z + t_z \\ 1 \end{pmatrix}$$

■ Rotation

■ z Axis Rotation Yaw - $R_z[\psi]$

$$\begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \\ 1 \end{pmatrix} = \begin{pmatrix} \cos[\theta] & -\sin[\theta] & 0 & 0 \\ \sin[\theta] & \cos[\theta] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

■ y Axis Rotation Pitch - $R_y[\theta]$

$$\begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \\ 1 \end{pmatrix} = \begin{pmatrix} \cos[\theta] & 0 & \sin[\theta] & 0 \\ 0 & 1 & 0 & 0 \\ -\sin[\theta] & 0 & \cos[\theta] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

■ x Axis Rotation Roll - $R_x[\phi]$

$$\begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos[\theta] & -\sin[\theta] & 0 \\ 0 & \sin[\theta] & \cos[\theta] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

■ Derive

$$\{x, y\} = \{r \cos[\phi], r \sin[\phi]\}$$

$$\{x', y'\} = \{r \cos[\phi + \theta], r \sin[\phi + \theta]\}$$

$$\{x', y'\} = \{r \cos[\phi + \theta], r \sin[\phi + \theta]\}$$

$$= \{r \cos[\phi] \cos[\theta] - r \sin[\phi] \sin[\theta], r \cos[\phi] \sin[\theta] + r \sin[\phi] \cos[\theta]\}$$

$$= \{x \cos[\theta] - y \sin[\theta], x \sin[\theta] + y \cos[\theta]\}$$

$$= \begin{pmatrix} \cos[\theta] & -\sin[\theta] \\ \sin[\theta] & \cos[\theta] \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix}$$

■ Transport $\begin{matrix} \text{ESC} & \text{tr} & \text{ESC} \end{matrix}$

$$\begin{pmatrix} a & b & c \end{pmatrix}^T = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

■ Matrix Property

$$\begin{pmatrix} A1 & A2 & A3 \\ A4 & A5 & A6 \\ A7 & A8 & A9 \end{pmatrix} \cdot \begin{pmatrix} B1 & B2 & B3 \\ B4 & B5 & B6 \\ B7 & B8 & B9 \end{pmatrix} =$$

$$\begin{pmatrix} A1 B1 + A2 B4 + A3 B7 & A1 B2 + A2 B5 + A3 B8 & A1 B3 + A2 B6 + A3 B9 \\ A4 B1 + A5 B4 + A6 B7 & A4 B2 + A5 B5 + A6 B8 & A4 B3 + A5 B6 + A6 B9 \\ A7 B1 + A8 B4 + A9 B7 & A7 B2 + A8 B5 + A9 B8 & A7 B3 + A8 B6 + A9 B9 \end{pmatrix}$$

$$\begin{pmatrix} A1 & A2 & A3 \\ A4 & A5 & A6 \\ A7 & A8 & A9 \end{pmatrix} * \begin{pmatrix} B1 & B2 & B3 \\ B4 & B5 & B6 \\ B7 & B8 & B9 \end{pmatrix} = \begin{pmatrix} A1 B1 & A2 B2 & A3 B3 \\ A4 B4 & A5 B5 & A6 B6 \\ A7 B7 & A8 B8 & A9 B9 \end{pmatrix}$$

■ Form

```
// MatrixForm
Norm[v] // TraditionalForm
||v|| // StandardForm
// FullForm
```

$\alpha = (a \ b \ c);$

$\alpha[[1, 1]] = a$

Determinant (행렬식)

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \text{Det} \left[\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \right] = a_{11} a_{22} - a_{12} a_{21}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \text{Det} \left[\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \right]$$

$$= a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$= a_{11} (a_{22} a_{33} - a_{23} a_{32}) - a_{12} (a_{21} a_{33} - a_{23} a_{31}) + a_{13} (a_{21} a_{32} - a_{22} a_{31})$$

Vector

```
{a, b, c} . {x, y, z} = Dot[{a, b, c}, {x, y, z}] = a x + b y + c z;
{a, b, c} * {x, y, z} = Times[{a, b, c}, {x, y, z}] = {a x, b y, c z};
{a, b, c} x {x, y, z} = Cross[{a, b, c}, {x, y, z}] = {-c y + b z, c x - a z, -b x + a y};
||{a, b, c}|| = Norm[{a, b, c}] =  $\sqrt{|a|^2 + |b|^2 + |c|^2}$ 
Power[{a, b, c}, {x, y, z}] = {ax, by, cz}
```

■ Vector Angle

$$\text{Cos}[\theta] = \frac{\mathbf{u} \cdot \mathbf{v}}{\text{Norm}[\mathbf{u}] \text{Norm}[\mathbf{v}]}$$

■ Vector $\langle \mathbf{U}, \mathbf{V} \rangle$ (`[ESC]*[ESC]`)

$$\langle \mathbf{U}, \mathbf{V} \rangle = \{u_1, u_2, u_3\} \times \{v_1, v_2, v_3\} = \{u_1 v_1, u_2 v_2, u_3 v_3\}$$

■ Vector Inner Product (Dot[{u₁, u₂, u₃},{v₁, v₂, v₃}], |U||V|Cos[θ])

$$\mathbf{U} \cdot \mathbf{V} = \{u_1, u_2, u_3\} \cdot \{v_1, v_2, v_3\} = u_1 v_1 + u_2 v_2 + u_3 v_3$$

■ Vector External Product (Cross[{u₁, u₂, u₃},{v₁, v₂, v₃}], `ESC`cross`ESC`)

$$\begin{aligned}
 \mathbf{U} \times \mathbf{V} &= \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\} \times \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\} = -\mathbf{V} \times \mathbf{U} \\
 &= \begin{vmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3 \\ \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 \\ \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 \end{vmatrix} = \text{Det} \left[\begin{pmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3 \\ \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 \\ \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 \end{pmatrix} \right] \\
 &= \mathbf{e}_1 (\mathbf{u}_2 \mathbf{v}_3 - \mathbf{u}_3 \mathbf{v}_2) - \mathbf{e}_2 (\mathbf{u}_1 \mathbf{v}_3 - \mathbf{u}_3 \mathbf{v}_1) + \mathbf{e}_3 (\mathbf{u}_1 \mathbf{v}_2 - \mathbf{u}_2 \mathbf{v}_1) \\
 &= \{\mathbf{u}_2 \mathbf{v}_3 - \mathbf{u}_3 \mathbf{v}_2, -\mathbf{u}_1 \mathbf{v}_3 + \mathbf{u}_3 \mathbf{v}_1, \mathbf{u}_1 \mathbf{v}_2 - \mathbf{u}_2 \mathbf{v}_1\}
 \end{aligned}$$

■ Vector Property

$$\begin{aligned}
 \mathbf{U} \times (\mathbf{V} \times \mathbf{W}) &= \langle \mathbf{U}, \mathbf{W} \rangle \mathbf{V} - \langle \mathbf{U}, \mathbf{V} \rangle \mathbf{W} \\
 \langle \mathbf{U}, \mathbf{V} \times \mathbf{W} \rangle &= \begin{vmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 \\ \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 \\ \mathbf{w}_1 & \mathbf{w}_2 & \mathbf{w}_3 \end{vmatrix} \\
 &= \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\} \times (\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\} \times \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\}) \\
 &= \{\mathbf{u}_1 (\mathbf{v}_2 \mathbf{w}_3 - \mathbf{v}_3 \mathbf{w}_2), \mathbf{u}_2 (-\mathbf{v}_1 \mathbf{w}_3 + \mathbf{v}_3 \mathbf{w}_1), \mathbf{u}_3 (\mathbf{v}_1 \mathbf{w}_2 - \mathbf{v}_2 \mathbf{w}_1)\}
 \end{aligned}$$

Area

$$\text{Area} = \frac{1}{2} \text{Det} \left[\begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix} \right] = \frac{x_3 (y_1 - y_2) + x_1 (y_2 - y_3) + x_2 (-y_1 + y_3)}{2}$$

■ Line & Point (Up|Down)

$$\text{Det} \left[\begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix} \right] = x_3 (y_1 - y_2) + x_1 (y_2 - y_3) + x_2 (-y_1 + y_3)$$

$$\text{Det} \left[\begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix} \right] < 0 : \text{Right Turn}$$

$$\text{Det} \left[\begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix} \right] > 0 : \text{Left Turn}$$

Rotation Matrix

$$\mathbf{R}_x[\phi] := \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos[\phi] & -\sin[\phi] \\ 0 & \sin[\phi] & \cos[\phi] \end{pmatrix} // \text{MatrixForm}$$

$$\mathbf{R}_y[\theta] := \begin{pmatrix} \cos[\theta] & 0 & \sin[\theta] \\ 0 & 1 & 0 \\ -\sin[\theta] & 0 & \cos[\theta] \end{pmatrix} // \text{MatrixForm}$$

$$\mathbf{R}_z[\psi] := \begin{pmatrix} \cos[\psi] & -\sin[\psi] & 0 \\ \sin[\psi] & \cos[\psi] & 0 \\ 0 & 0 & 1 \end{pmatrix} // \text{MatrixForm}$$

■ Euler Angle

$$\mathbf{R} = \mathbf{R}_z[\psi] \cdot \mathbf{R}_y[\theta] \cdot \mathbf{R}_x[\phi] = \begin{pmatrix} \cos[\theta] \cos[\psi] & \cos[\psi] \sin[\theta] \sin[\phi] - \cos[\phi] \sin[\psi] & \cos[\phi] \cos[\psi] \\ \cos[\theta] \sin[\psi] & \cos[\phi] \cos[\psi] + \sin[\theta] \sin[\phi] \sin[\psi] & -\cos[\psi] \sin[\phi] \\ -\sin[\theta] & \cos[\theta] \sin[\phi] & \cos[\theta] \cos[\phi] \end{pmatrix}$$

$$\mathbf{R}_z[\psi] \cdot \mathbf{R}_y[\theta] \cdot \mathbf{R}_x[\phi]$$

$$\begin{pmatrix} \cos[\psi] & -\sin[\psi] & 0 \\ \sin[\psi] & \cos[\psi] & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos[\theta] & 0 & \sin[\theta] \\ 0 & 1 & 0 \\ -\sin[\theta] & 0 & \cos[\theta] \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos[\phi] & -\sin[\phi] \\ 0 & \sin[\phi] & \cos[\phi] \end{pmatrix} // \text{MatrixForm}$$

$$\begin{pmatrix} \cos[\theta] \cos[\psi] & \cos[\psi] \sin[\theta] \sin[\phi] - \cos[\phi] \sin[\psi] & \cos[\phi] \cos[\psi] \sin[\theta] + \sin[\phi] \sin[\psi] \\ \cos[\theta] \sin[\psi] & \cos[\phi] \cos[\psi] + \sin[\theta] \sin[\phi] \sin[\psi] & -\cos[\psi] \sin[\phi] + \cos[\phi] \sin[\theta] \sin[\psi] \\ -\sin[\theta] & \cos[\theta] \sin[\phi] & \cos[\theta] \cos[\phi] \end{pmatrix}$$

$$\begin{pmatrix} \cos[\psi] & \sin[\psi] & 0 \\ -\sin[\psi] & \cos[\psi] & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos[\theta] & 0 & -\sin[\theta] \\ 0 & 1 & 0 \\ \sin[\theta] & 0 & \cos[\theta] \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos[\phi] & \sin[\phi] \\ 0 & -\sin[\phi] & \cos[\phi] \end{pmatrix} // \text{MatrixForm}$$

$$\begin{pmatrix} \cos[\theta] \cos[\psi] & \cos[\psi] \sin[\theta] \sin[\phi] + \cos[\phi] \sin[\psi] & -\cos[\phi] \cos[\psi] \sin[\theta] + \sin[\phi] \sin[\psi] \\ -\cos[\theta] \sin[\psi] & \cos[\phi] \cos[\psi] - \sin[\theta] \sin[\phi] \sin[\psi] & \cos[\psi] \sin[\phi] + \cos[\phi] \sin[\theta] \sin[\psi] \\ \sin[\theta] & -\cos[\theta] \sin[\phi] & \cos[\theta] \cos[\phi] \end{pmatrix}$$

Rotation matrix from axis and angle

$$\mathbf{R}[\theta] :=$$

$$\begin{pmatrix} \cos[\theta] + u_x^2 (1 - \cos[\theta]) & u_x u_y (1 - \cos[\theta]) - u_z \sin[\theta] & u_x u_z (1 - \cos[\theta]) + u_y \sin[\theta] \\ u_y u_x (1 - \cos[\theta]) + u_z \sin[\theta] & \cos[\theta] + u_y^2 (1 - \cos[\theta]) & u_y u_z (1 - \cos[\theta]) - u_x \sin[\theta] \\ u_z u_x (1 - \cos[\theta]) - u_y \sin[\theta] & u_z u_y (1 - \cos[\theta]) + u_x \sin[\theta] & \cos[\theta] + u_z^2 (1 - \cos[\theta]) \end{pmatrix}$$

■ concisely

$$\mathbf{R} = \mathbf{I} \cdot \cos[\theta] + \sin[\theta] \mathbf{u}_x + (1 - \cos[\theta]) \mathbf{u} \otimes \mathbf{u}$$

$$\mathbf{I} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{u}_x = \begin{pmatrix} 0 & -u_z & u_y \\ u_z & 0 & -u_x \\ -u_y & u_x & 0 \end{pmatrix}$$

$$\mathbf{u} \otimes \mathbf{u} = \begin{pmatrix} u_x^2 & u_x u_y & u_x u_z \\ u_x u_y & u_y^2 & u_y u_z \\ u_x u_z & u_y u_z & u_z^2 \end{pmatrix}$$

■ ex

```
{ux, uy, uz} = {0, 1, 0};


$$\begin{pmatrix} \cos[\theta] + u_x^2 (1 - \cos[\theta]) & u_x u_y (1 - \cos[\theta]) - u_z \sin[\theta] & u_x u_z (1 - \cos[\theta]) + u_y \sin[\theta] \\ u_y u_x (1 - \cos[\theta]) + u_z \sin[\theta] & \cos[\theta] + u_y^2 (1 - \cos[\theta]) & u_y u_z (1 - \cos[\theta]) - u_x \sin[\theta] \\ u_z u_x (1 - \cos[\theta]) - u_y \sin[\theta] & u_z u_y (1 - \cos[\theta]) + u_x \sin[\theta] & \cos[\theta] + u_z^2 (1 - \cos[\theta]) \end{pmatrix}$$


// MatrixForm


$$\begin{pmatrix} \cos[\theta] & 0 & \sin[\theta] \\ 0 & 1 & 0 \\ -\sin[\theta] & 0 & \cos[\theta] \end{pmatrix}$$

```

Direction

```

$$\begin{pmatrix} 1 & 0 & 0 & POx \\ 0 & 1 & 0 & POy \\ 0 & 0 & 1 & POz \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos[Zh] & -\sin[Zh] & 0 & 0 \\ \sin[Zh] & \cos[Zh] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos[Yh] & 0 & \sin[Yh] & 0 \\ 0 & 1 & 0 & 0 \\ -\sin[Yh] & 0 & \cos[Yh] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos[Xh] & -\sin[Xh] & 0 \\ 0 & \sin[Xh] & \cos[Xh] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}^T[[1]] // FullSimplify$$

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```

$$\begin{pmatrix} 1 & 0 & 0 & POx \\ 0 & 1 & 0 & POy \\ 0 & 0 & 1 & POz \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos[Zh] & -\sin[Zh] & 0 & 0 \\ \sin[Zh] & \cos[Zh] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos[Yh] & 0 & \sin[Yh] & 0 \\ 0 & 1 & 0 & 0 \\ -\sin[Yh] & 0 & \cos[Yh] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos[Xh] & -\sin[Xh] & 0 \\ 0 & \sin[Xh] & \cos[Xh] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}^T[[1]] // FullSimplify$$

```

```

$$\begin{pmatrix} 1 & 0 & 0 & POx \\ 0 & 1 & 0 & POy \\ 0 & 0 & 1 & POz \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos[Zh] & -\sin[Zh] & 0 & 0 \\ \sin[Zh] & \cos[Zh] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos[Yh] & 0 & \sin[Yh] & 0 \\ 0 & 1 & 0 & 0 \\ -\sin[Yh] & 0 & \cos[Yh] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos[Xh] & -\sin[Xh] & 0 \\ 0 & \sin[Xh] & \cos[Xh] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}^T[[1]] // FullSimplify$$

```

```
{POx + Cos[Yh] Cos[Zh], POy + Cos[Yh] Sin[Zh], POz - Sin[Yh], 1}
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```
{POx + Cos[Zh] Sin[Xh] Sin[Yh] - Cos[Xh] Sin[Zh],  
POy + Cos[Xh] Cos[Zh] + Sin[Xh] Sin[Yh] Sin[Zh], POz + Cos[Yh] Sin[Xh], 1}
```

```
{POx + Cos[Xh] Cos[Zh] Sin[Yh] + Sin[Xh] Sin[Zh],  
POy - Cos[Zh] Sin[Xh] + Cos[Xh] Sin[Yh] Sin[Zh], POz + Cos[Xh] Cos[Yh], 1}
```

Multiple

$$\begin{pmatrix} DX & 0 & 0 & POx \\ 0 & DX & 0 & POy \\ 0 & 0 & DX & POz \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos[Zh] & -\sin[Zh] & 0 & 0 \\ \sin[Zh] & \cos[Zh] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos[Yh] & 0 & \sin[Yh] & 0 \\ 0 & 1 & 0 & 0 \\ -\sin[Yh] & 0 & \cos[Yh] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos[Xh] & -\sin[Xh] & 0 \\ 0 & \sin[Xh] & \cos[Xh] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}^T[[1]] // \text{FullSimplify}$$

$$\begin{pmatrix} DY & 0 & 0 & POx \\ 0 & DY & 0 & POy \\ 0 & 0 & DY & POz \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos[Zh] & -\sin[Zh] & 0 & 0 \\ \sin[Zh] & \cos[Zh] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos[Yh] & 0 & \sin[Yh] & 0 \\ 0 & 1 & 0 & 0 \\ -\sin[Yh] & 0 & \cos[Yh] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos[Xh] & -\sin[Xh] & 0 \\ 0 & \sin[Xh] & \cos[Xh] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}^T[[1]] // \text{FullSimplify}$$

$$\begin{pmatrix} DZ & 0 & 0 & POx \\ 0 & DZ & 0 & POy \\ 0 & 0 & DZ & POz \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos[Zh] & -\sin[Zh] & 0 & 0 \\ \sin[Zh] & \cos[Zh] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos[Yh] & 0 & \sin[Yh] & 0 \\ 0 & 1 & 0 & 0 \\ -\sin[Yh] & 0 & \cos[Yh] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos[Xh] & -\sin[Xh] & 0 \\ 0 & \sin[Xh] & \cos[Xh] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}^T[[1]] // \text{FullSimplify}$$

{POx + DX Cos[Yh] Cos[Zh], POy + DX Cos[Yh] Sin[Zh], POz - DX Sin[Yh], 1}

{POx + DY Cos[Zh] Sin[Xh] Sin[Yh] - DY Cos[Xh] Sin[Zh],
POy + DY Cos[Xh] Cos[Zh] + DY Sin[Xh] Sin[Yh] Sin[Zh], POz + DY Cos[Yh] Sin[Xh], 1}

{POx + DZ Cos[Xh] Cos[Zh] Sin[Yh] + DZ Sin[Xh] Sin[Zh],
POy - DZ Cos[Zh] Sin[Xh] + DZ Cos[Xh] Sin[Yh] Sin[Zh], POz + DZ Cos[Xh] Cos[Yh], 1}

$$\begin{pmatrix} 1 & 0 & 0 & P0x \\ 0 & 1 & 0 & P0y \\ 0 & 0 & 1 & P0z \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos[Zh] & -\sin[Zh] & 0 & 0 \\ \sin[Zh] & \cos[Zh] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos[Yh] & 0 & \sin[Yh] & 0 \\ 0 & 1 & 0 & 0 \\ -\sin[Yh] & 0 & \cos[Yh] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos[Xh] & -\sin[Xh] & 0 \\ 0 & \sin[Xh] & \cos[Xh] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} DX \\ 0 \\ 0 \\ 1 \end{pmatrix}^T[[1]] // \text{FullSimplify}$$

$$\begin{pmatrix} 1 & 0 & 0 & P0x \\ 0 & 1 & 0 & P0y \\ 0 & 0 & 1 & P0z \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos[Zh] & -\sin[Zh] & 0 & 0 \\ \sin[Zh] & \cos[Zh] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos[Yh] & 0 & \sin[Yh] & 0 \\ 0 & 1 & 0 & 0 \\ -\sin[Yh] & 0 & \cos[Yh] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos[Xh] & -\sin[Xh] & 0 \\ 0 & \sin[Xh] & \cos[Xh] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ DY \\ 0 \\ 1 \end{pmatrix}^T[[1]] // \text{FullSimplify}$$

$$\begin{pmatrix} 1 & 0 & 0 & P0x \\ 0 & 1 & 0 & P0y \\ 0 & 0 & 1 & P0z \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos[Zh] & -\sin[Zh] & 0 & 0 \\ \sin[Zh] & \cos[Zh] & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos[Yh] & 0 & \sin[Yh] & 0 \\ 0 & 1 & 0 & 0 \\ -\sin[Yh] & 0 & \cos[Yh] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos[Xh] & -\sin[Xh] & 0 \\ 0 & \sin[Xh] & \cos[Xh] & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ DZ \\ 1 \end{pmatrix}^T[[1]] // \text{FullSimplify}$$

$$\{P0x + DX \cos[Yh] \cos[Zh], P0y + DX \cos[Yh] \sin[Zh], P0z - DX \sin[Yh], 1\}$$

$$\{P0x + DY \cos[Zh] \sin[Xh] \sin[Yh] - DY \cos[Xh] \sin[Zh], \\ P0y + DY \cos[Xh] \cos[Zh] + DY \sin[Xh] \sin[Yh] \sin[Zh], P0z + DY \cos[Yh] \sin[Xh], 1\}$$

$$\{P0x + DZ \cos[Xh] \cos[Zh] \sin[Yh] + DZ \sin[Xh] \sin[Zh], \\ P0y - DZ \cos[Zh] \sin[Xh] + DZ \cos[Xh] \sin[Yh] \sin[Zh], P0z + DZ \cos[Xh] \cos[Yh], 1\}$$

$$dX = \sqrt{PXx^2 + PXy^2}$$

$$Zh = \text{ArcTan}[PXx, PXy]$$

$$\tan[Zh] = \frac{PXy}{PXx} = \frac{\frac{PXy}{dX}}{\frac{PXx}{dX}} = \frac{\sin[Zh]}{\cos[Zh]}$$

$$dX = \sqrt{PXx^2 + PXy^2}$$

$$dX^2 = PXx^2 + PXy^2$$

$$dX = \frac{PXx^2 + PXy^2}{dX} = \frac{PXx^2}{dX} + \frac{PXy^2}{dX} = \frac{PXx}{dX} PXx + \frac{PXy}{dX} PXy \\ = \cos[Zh] PXx + \sin[Zh] PXy$$

$$DX = \sqrt{PXx^2 + PXy^2 + PXz^2}$$

$$Yh = \text{ArcTan}[\cos[Zh] PXx + \sin[Zh] PXy, -PXz]$$

$$\begin{aligned}
\tan[Y_h] &= \frac{-PX_z}{dX} = \frac{\frac{-PX_z}{dX}}{\frac{dX}{dX}} = \frac{\sin[Y_h]}{\cos[Y_h]} \\
DX &= \sqrt{dX^2 + (-PX_z)^2} \\
DX^2 &= dX^2 + (-PX_z)^2 \\
DX &= \frac{dX^2 + (-PX_z)^2}{DX} = \frac{dX^2}{DX} + \frac{(-PX_z)^2}{DX} = \frac{dX}{DX} dX + \frac{(-PX_z)}{DX} (-PX_z) \\
&= \cos[Y_h] dX + \sin[Y_h] (-PX_z) \\
&= \cos[Y_h] \left\{ \frac{PX_x^2 + PY^2}{dX} \right\} + \sin[Y_h] (-PX_z) \\
&= \cos[Y_h] \left\{ \frac{PX_x}{dX} PX_x + \frac{PY}{dX} PY \right\} + \sin[Y_h] (-PX_z) \\
&= \cos[Y_h] \{ \cos[Z_h] PX_x + \sin[Z_h] PY \} + \sin[Y_h] (-PX_z) \\
&= \cos[Y_h] \cos[Z_h] PX_x + \cos[Y_h] \sin[Z_h] PY + \sin[Y_h] (-PX_z) \\
&= \cos[Y_h] \cos[Z_h] PX_x + \cos[Y_h] \sin[Z_h] PY - \sin[Y_h] PX_z
\end{aligned}$$

Distance

$$\begin{aligned}
Z_h &= \text{ArcTan}[x, y] \\
Y_h &= \text{ArcTan}[\cos[Z_h] x + \sin[Z_h] y, z] \\
D &= \sqrt{d^2 + z^2} \\
D^2 &= d^2 + z^2 \\
D &= \frac{d^2 + z^2}{D} = \frac{d^2}{D} + \frac{z^2}{D} = \frac{d}{D} d + \frac{z}{D} z \\
&= \cos[Y_h] d + \sin[Y_h] z \\
&= \cos[Y_h] \left\{ \frac{x^2 + y^2}{d} \right\} + \sin[Y_h] z \\
&= \cos[Y_h] \left\{ \frac{x}{d} x + \frac{y}{d} y \right\} + \sin[Y_h] z \\
&= \cos[Y_h] \{ \cos[Z_h] x + \sin[Z_h] y \} + \sin[Y_h] z \\
&= \cos[Y_h] \cos[Z_h] x + \cos[Y_h] \sin[Z_h] y + \sin[Y_h] z
\end{aligned}$$

■ ∴

$$\{P0x + D \cos[Y_h] \cos[Z_h], P0y + D \cos[Y_h] \sin[Z_h], P0z - D \sin[Y_h]\}$$

$$\begin{aligned}
Z_h &= \text{ArcTan}[x, y] \\
Y_h &= \text{ArcTan}[\cos[Z_h] x + \sin[Z_h] y, z] \\
D &= \cos[Y_h] \cos[Z_h] x + \cos[Y_h] \sin[Z_h] y + \sin[Y_h] z
\end{aligned}$$

Ex. Formula and solve

■ 1

$$\{d \cos[\theta] \cos[\varphi], d \cos[\theta] \sin[\varphi], d \sin[\theta]\} = \{x, y, z\}$$

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d = (x - x0) / Cos[θ] Cos[φ]
d = (y - y0) / Cos[φ] Sin[θ]
d = (z - z0) / Sin[φ]

S = Simplify[
  Solve[{(x - x0) / Cos[θ] Cos[φ] == (y - y0) / Cos[φ] Sin[θ] == (z - z0) / Sin[φ], z == h},
    {x, y, z}][[1]]];
{x,
 y,
 z} /.
S

```

■ 2

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Simplify[Transpose[
  
$$\begin{pmatrix} \cos[\theta] & -\sin[\theta] & 0 \\ \sin[\theta] & \cos[\theta] & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos[\phi] & 0 & -\sin[\phi] \\ 0 & 1 & 0 \\ \sin[\phi] & 0 & \cos[\phi] \end{pmatrix} \cdot \begin{pmatrix} d \\ d \tan[\phi] \cos[t] \\ d \tan[\phi] \sin[t] \end{pmatrix}][[1]] + \{x0, y0, z0\}$$

  {x0 + d (-Cos[t] Sin[θ] Tan[φ] + Cos[θ] (Cos[φ] - Sin[t] Sin[φ] Tan[φ])),
    y0 + d (Cos[φ] Sin[θ] + (Cos[t] Cos[θ] - Sin[t] Sin[θ] Sin[φ]) Tan[φ]),
    z0 + d (Sin[φ] + Cos[φ] Sin[t] Tan[φ])} = {x, y, z}

d = (x - x0) / (-Cos[t] Sin[θ] Tan[φ] + Cos[θ] (Cos[φ] - Sin[t] Sin[φ] Tan[φ]))
d = (y - y0) / (Cos[φ] Sin[θ] + (Cos[t] Cos[θ] - Sin[t] Sin[θ] Sin[φ]) Tan[φ])
d = (z - z0) / (Sin[φ] + Cos[φ] Sin[t] Tan[φ])

S = Solve[{(x - x0) / (-Cos[t] Sin[θ] Tan[φ] + Cos[θ] (Cos[φ] - Sin[t] Sin[φ] Tan[φ]))
  == (y - y0) / (Cos[φ] Sin[θ] + (Cos[t] Cos[θ] - Sin[t] Sin[θ] Sin[φ]) Tan[φ])
  == (z - z0) / (Sin[φ] + Cos[φ] Sin[t] Tan[φ]), y == h}, {x, y, z}][[1]] // Simplify //
  InputForm;
{x // InputForm, y // InputForm, z // InputForm} /. S[[1]]

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■ 3

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Simplify[Transpose[
  
$$\begin{pmatrix} \cos[\theta] & -\sin[\theta] & 0 \\ \sin[\theta] & \cos[\theta] & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos[\phi] & 0 & -\sin[\phi] \\ 0 & 1 & 0 \\ \sin[\phi] & 0 & \cos[\phi] \end{pmatrix} \cdot \begin{pmatrix} d \\ d \tan[\phi] \\ 0 \end{pmatrix}][[1]] + \{x0, y0, z0\}$$

  {x0, y0, z0}

Simplify[
  Transpose[
    
$$\begin{pmatrix} \cos[\theta] & -\sin[\theta] & 0 \\ \sin[\theta] & \cos[\theta] & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos[\phi] & 0 & -\sin[\phi] \\ 0 & 1 & 0 \\ \sin[\phi] & 0 & \cos[\phi] \end{pmatrix} \cdot \begin{pmatrix} D \\ 0 \\ D \tan[\phi] \end{pmatrix}][[1]] + \{x0, y0, z0\}$$

  ]

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■ 4

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{Cos[ThH] Cos[JhH], Cos[JhH] Sin[ThH], Sin[JhH]}

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Simplify[Transpose[
  
$$\begin{pmatrix} \cos[\theta] & -\sin[\theta] & 0 \\ \sin[\theta] & \cos[\theta] & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos[\varphi] & 0 & -\sin[\varphi] \\ 0 & 1 & 0 \\ \sin[\varphi] & 0 & \cos[\varphi] \end{pmatrix} \cdot \begin{pmatrix} d \\ d \tan[\phi] \cos[t] \\ d \tan[\phi] \sin[t] \end{pmatrix}][[1]] + \{x0, y0, z0\} \\
\{x0 + d (-\cos[t] \sin[\theta] \tan[\phi] + \cos[\theta] (\cos[\varphi] - \sin[t] \sin[\varphi] \tan[\phi])), \\
y0 + d (\cos[\varphi] \sin[\theta] + (\cos[t] \cos[\theta] - \sin[t] \sin[\theta] \sin[\varphi]) \tan[\phi]), \\
z0 + d (\sin[\varphi] + \cos[\varphi] \sin[t] \tan[\phi])\} = \{x, y, z\} \\
d = (x - x0) / (-\cos[t] \sin[\theta] \tan[\phi] + \cos[\theta] (\cos[\varphi] - \sin[t] \sin[\varphi] \tan[\phi])) \\
d = (y - y0) / (\cos[\varphi] \sin[\theta] + (\cos[t] \cos[\theta] - \sin[t] \sin[\theta] \sin[\varphi]) \tan[\phi]) \\
d = (z - z0) / (\sin[\varphi] + \cos[\varphi] \sin[t] \tan[\phi]) \\
S = Solve[\{(x - x0) / (-\cos[t] \sin[\theta] \tan[\phi] + \cos[\theta] (\cos[\varphi] - \sin[t] \sin[\varphi] \tan[\phi])) \\
== (y - y0) / (\cos[\varphi] \sin[\theta] + (\cos[t] \cos[\theta] - \sin[t] \sin[\theta] \sin[\varphi]) \tan[\phi]) \\
== (z - z0) / (\sin[\varphi] + \cos[\varphi] \sin[t] \tan[\phi])\}, \\
Cos[ThH] Cos[JhH] (x - xh0) + Cos[JhH] Sin[ThH] (y - yh0) + Sin[JhH] (z - zh0) == 0\}, \\
\{x, y, z\}][[1]] // Simplify // InputForm; \\
\{x // InputForm, y // InputForm, z // InputForm\} /. S[[1]]$$

```

Low of Cosine

$$a^2 = b^2 + c^2 - 2 b c \cos[A]$$

$$\cos[a] = \cos[b] \cos[c] + \sin[b] \sin[c] \cos[A]$$

Dot[{a_x, a_y}, {b_x, b_y}]

$$A = \{a_x, a_y\}$$

$$B = \{b_x, b_y\}$$

$$A \cdot B = a_x b_x + a_y b_y$$

$$\text{Dot}[A, B] == \text{Norm}[A] \text{Norm}[B] \cos[\theta]$$

$$A_B = \text{Norm}[A] \cos[\theta]$$

$$A_B = A \cdot \hat{B}$$

$$\hat{B} = \frac{B}{\text{Norm}[B]} \quad \{ \therefore \text{unit vector} \}$$