

Path Following Circular Arc

$$x = r \cos[\phi]$$

$$y = r \sin[\phi]$$

$$r = \sqrt{x^2 + y^2}$$

$$\phi = \text{ArcTan}[x, y] = \text{ArcTan}\left[\frac{y}{x}\right]$$

$$\sin[a \pm b] = \sin[a] \cos[b] \pm \sin[b] \cos[a]$$

$$\sin[2a] = 2 \sin[a] \cos[a]$$

$$\cos[a \pm b] = \cos[a] \cos[b] \mp \sin[a] \sin[b]$$

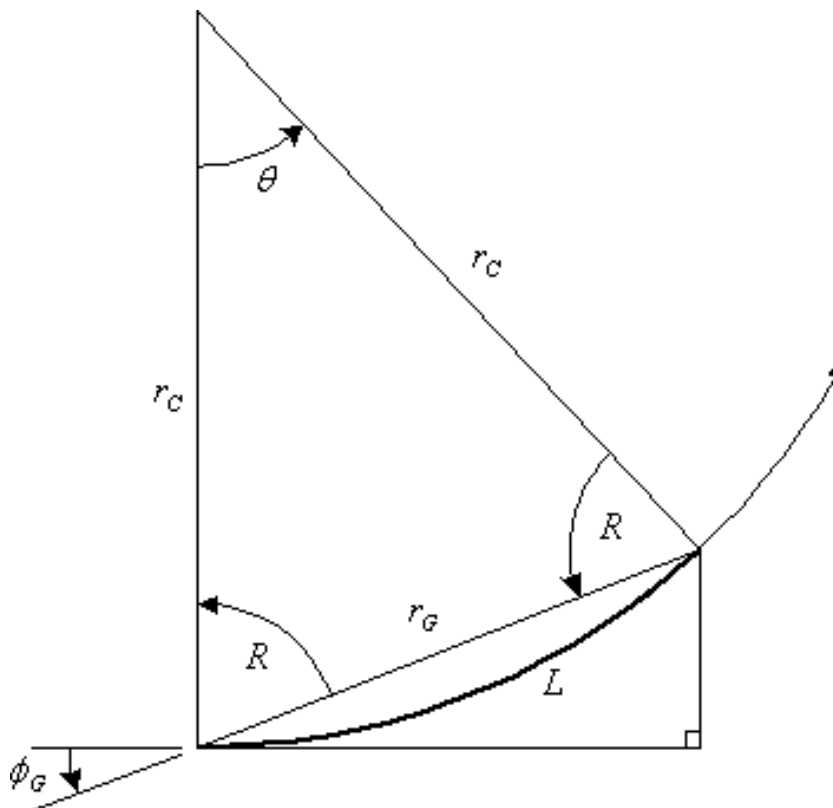
$$\sin\left[a + \frac{\pi}{2}\right] = \cos[a]$$

$$\sin\left[a - \frac{\pi}{2}\right] = -\cos[a]$$

$$\cos\left[a + \frac{\pi}{2}\right] = -\sin[a]$$

$$\cos\left[a - \frac{\pi}{2}\right] = \sin[a]$$

Trajectory based on an arc of a circle



$$\blacksquare \phi_G + R = \frac{\pi}{2}, \quad \theta + 2R = \pi$$

$$\therefore \theta = 2\phi_G$$

$$\blacksquare \frac{\sin[R]}{r_c} = \frac{\sin[\theta]}{r_G} = \frac{\sin[R]}{r_c}$$

$$\frac{\sin[\theta]}{r_G} = \frac{\sin[R]}{r_c}$$

$$\frac{\sin[2\phi_G]}{r_G} = \frac{\sin[\frac{\pi}{2} - \phi_G]}{r_c}$$

$$r_c = r_G \frac{\sin[\frac{\pi}{2} - \phi_G]}{\sin[2\phi_G]}$$

$$r_c = r_G \frac{\cos[\phi_G]}{2\sin[\phi_G]\cos[\phi_G]}$$

$$r_c = r_G \frac{\cos[\phi_G]}{2\sin[\phi_G]\cos[\phi_G]}$$

$$\therefore r_c = \frac{r_G}{2\sin[\phi_G]}$$

$$\blacksquare L = r_c \theta$$

$$\therefore L = \frac{r_G \cdot \phi_G}{\sin[\phi_G]}$$

Geometry in Motion

$$t_1 - t_0 = \frac{L}{s}$$

$$\omega = \frac{\theta}{t_1 - t_0}$$

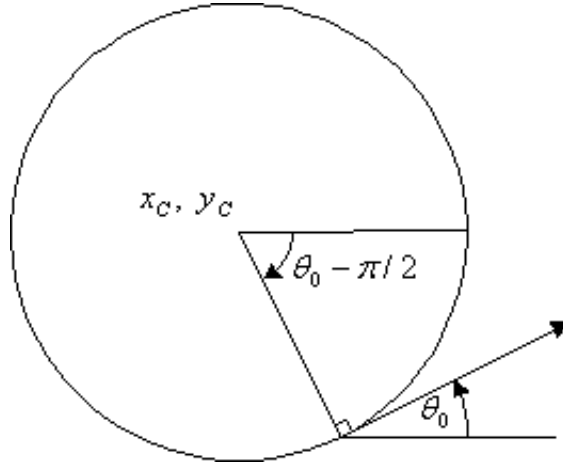
$$\theta_t = \omega (t - t_0) + \theta_0$$

Trigonometry at Work

$$x_t = x_0 - r_c \sin[\theta_0] + r_c \sin[\omega (t - t_0) + \theta_0]$$

$$y_t = y_0 + r_c \cos[\theta_0] - r_c \cos[\omega (t - t_0) + \theta_0]$$

the coordinates for center of the circle



■ left turn

$$x_c = x_0 + r_c \cos\left[\theta_0 + \frac{\pi}{2}\right]$$

$$y_c = y_0 + r_c \sin\left[\theta_0 + \frac{\pi}{2}\right]$$

■ right turn

$$x_c = x_0 - r_c \cos\left[\theta_0 - \frac{\pi}{2}\right]$$

$$y_c = y_0 - r_c \sin\left[\theta_0 - \frac{\pi}{2}\right]$$

■ $\therefore$

$$x_c = x_0 - r_c \sin[\theta_0]$$

$$y_c = y_0 + r_c \cos[\theta_0]$$

■ $\theta_t = \omega (t - t_0) + \theta_0$

$$x_c = x_t + r_c \cos\left[\theta_t + \frac{\pi}{2}\right]$$

$$y_c = y_t + r_c \sin\left[\theta_t + \frac{\pi}{2}\right]$$

$$x_c = x_t - r_c \cos\left[\theta_t - \frac{\pi}{2}\right]$$

$$y_c = y_t - r_c \sin\left[\theta_t - \frac{\pi}{2}\right]$$

$$x_t = x_c + r_c \cos\left[\theta_t - \frac{\pi}{2}\right]$$

$$y_t = y_c + r_c \sin\left[\theta_t - \frac{\pi}{2}\right]$$

$$x_t = x_c + r_c \cos\left[\omega (t - t_0) + \theta_0 - \frac{\pi}{2}\right]$$

$$y_t = y_c + r_c \sin\left[\omega (t - t_0) + \theta_0 - \frac{\pi}{2}\right]$$

■ $\therefore$

$$x_t = x_c + r_c \sin[\omega (t - t_0) + \theta_0]$$

$$y_t = y_c - r_c \cos[\omega (t - t_0) + \theta_0]$$

■ $x_c = x_0 - r_c \sin[\theta_0]$

$y_c = y_0 + r_c \cos[\theta_0]$

$$x_t = x_0 - r_c \sin[\theta_0] + r_c \sin[\omega (t - t_0) + \theta_0]$$

$$y_t = y_0 + r_c \cos[\theta_0] - r_c \cos[\omega (t - t_0) + \theta_0]$$

Supplement

s : Speed, ω : Rotational velocity

■ $\theta_t = \omega (t - t_0) + \theta_0$

$$\frac{dx}{dt} = s \cos[\omega (t - t_0) + \theta_0], \quad \frac{dy}{dt} = s \sin[\omega (t - t_0) + \theta_0]$$

■ Integrating

$$x_t = \frac{s}{\omega} \sin[\omega (t - t_0) + \theta_0] + k_x$$

$$y_t = -\frac{s}{\omega} \cos[\omega (t - t_0) + \theta_0] + k_y$$

■ $t = t_0$

$$x_0 = \frac{s}{\omega} \sin[\theta_0] + k_x$$

$$y_0 = -\frac{s}{\omega} \cos[\theta_0] + k_y$$

$$k_x = x_0 - \frac{s}{\omega} \sin[\theta_0]$$

$$k_y = y_0 + \frac{s}{\omega} \cos[\theta_0]$$

$$\blacksquare r_c = \frac{S}{\omega}$$

$$x_t = x_0 - \frac{S}{\omega} \sin[\theta_0] + \frac{S}{\omega} \sin[\omega (t - t_0) + \theta_0]$$

$$y_t = y_0 + \frac{S}{\omega} \cos[\theta_0] - \frac{S}{\omega} \cos[\omega (t - t_0) + \theta_0]$$

Robot Position

$$\begin{aligned} \text{Odometer} & += \frac{\text{Left Encoder Diff} + \text{Right Encoder Diff}}{2} \\ \text{Goniometer} & += \frac{\text{Right Encoder Diff} - \text{Left Encoder Diff}}{\text{Wheel Base}} \end{aligned}$$

$$x_t = x_{t-1} + \text{Odometer} * \cos[\text{Goniometer}]$$

$$y_t = y_{t-1} + \text{Odometer} * \sin[\text{Goniometer}]$$

$$\blacksquare L = r_c \cdot \theta$$

$$x_t = x_0 - r_c \sin[\theta_0] + r_c \sin[\omega (t - t_0) + \theta_0]$$

$$y_t = y_0 + r_c \cos[\theta_0] - r_c \cos[\omega (t - t_0) + \theta_0]$$

$$\blacksquare x_t = x_0 - r_c \sin[\theta_0] + r_c \sin[\theta_t]$$

$$y_t = y_0 + r_c \cos[\theta_0] - r_c \cos[\theta_t]$$

$$x_c = x_0 - r_c \sin[\theta_0]$$

$$y_c = y_0 + r_c \cos[\theta_0]$$

$$x_t = x_c + r_c \sin[\theta_t]$$

$$y_t = y_c - r_c \cos[\theta_t]$$

$$\blacksquare \theta = \theta_t - \theta_0, \quad r_c = \frac{L}{\theta}$$

$$x_t = x_0 - \frac{L}{\theta} \sin[\theta_0] + \frac{L}{\theta} \sin[\theta + \theta_0]$$

$$y_t = y_0 + \frac{L}{\theta} \cos[\theta_0] - \frac{L}{\theta} \cos[\theta + \theta_0]$$

$$\blacksquare L, \theta_t$$

$$x_t = x_{t-1} + L * \cos[\theta_t]$$

$$y_t = y_{t-1} + L * \sin[\theta_t]$$