

Cubic Hermite Curves

■ Given

Starting and Ending Points and Tangents

$$P_0, u_0, P_1, u_1$$

■ General Form

$$P(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

$$\frac{d}{dt} P(t) = P'(t) = a_1 + 2 a_2 t + 3 a_3 t^2$$

■ Boundary Conditions

$$P(0) = P_0$$

$$P'(0) = u_0$$

$$P(1) = P_1$$

$$P'(1) = u_1$$

■ Solving for the a_i

$$a_0 = P_0$$

$$a_1 = u_0$$

$$a_2 = -3 a_0 - 2 a_1 + 3 P_1 - u_1 = 3 (P_1 - P_0) - 2 u_0 - u_1$$

$$a_3 = 2 a_0 + a_1 - 2 P_1 + u_1 = u_0 + u_1 - 2 (P_1 - P_0)$$

$$\text{Solve}[\{P_1 == a_0 + a_1 + a_2 + a_3, u_1 == a_1 + 2 a_2 + 3 a_3\}, \{a_2, a_3\}]$$

$$\{\{a_2 \rightarrow -3 a_0 - 2 a_1 + 3 P_1 - u_1, a_3 \rightarrow 2 a_0 + a_1 - 2 P_1 + u_1\}\}$$

$$\{\{a_2 \rightarrow -3 a_0 - 2 a_1 + 3 P_1 - u_1, a_3 \rightarrow 2 a_0 + a_1 - 2 P_1 + u_1\}\}$$

■ Substitutiing these a_i into the "general form" above

$$P(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 = P_0 + u_0 t + (3 (P_1 - P_0) - 2 u_0 - u_1) t^2 + (u_0 + u_1 - 2 (P_1 - P_0)) t^3$$

■ Rewriting

$$\mathbf{P}(t) = \begin{pmatrix} t^3 & t^2 & t & 1 \end{pmatrix} \begin{pmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{P}_0 \\ \mathbf{P}_1 \\ \mathbf{u}_0 \\ \mathbf{u}_1 \end{pmatrix}$$

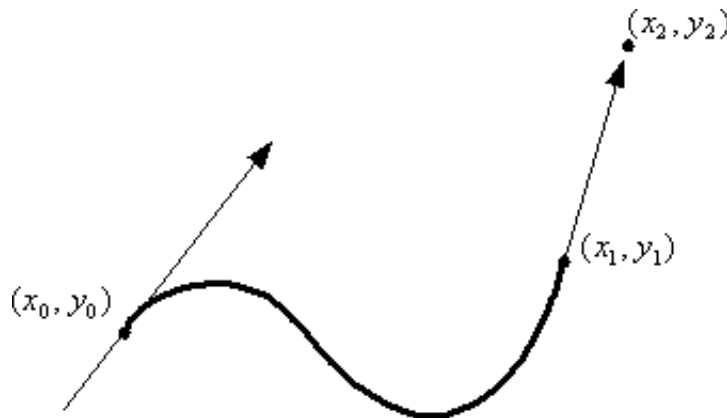
Multiplying with right association leads directly to the previous expression. Multiplying with left association leads to an equivalent but slightly different form

$$\mathbf{P}(t) = (1 - 3t^2 + 2t^3) \mathbf{P}_0 + (3t^2 - 2t^3) \mathbf{P}_1 + (t - 2t^2 + t^3) \mathbf{u}_0 + (-t^2 + t^3) \mathbf{u}_1$$

This is the Hermite Basis form.

Calculations Useful for Robotics

■ Hermit Curves



$$\begin{aligned} x_t &= A_x (t - t_0)^3 + B_x (t - t_0)^2 + C_x (t - t_0) + D_x \\ y_t &= A_y (t - t_0)^3 + B_y (t - t_0)^2 + C_y (t - t_0) + D_y \end{aligned}$$

■ Specified positions of robot

$$\begin{aligned} (x_0, y_0) \\ (x_1, y_1) \\ (x_2, y_2) \end{aligned}$$

■ Specified orientation of robot at the initial and final points of its first segment

$$\theta_0, \theta_1$$

■ Specified speed of robot initial and final points of its first segment

$$s_0, s_1$$

- Specified time that robot reaches initial and final points of its first segment

$$t_0, t_1$$

- Time variable

$$t$$

- Derived, third-degree polynomials giving x and y-coordinates of the robot's position as a function of time

$$x_t, y_t$$

- Derived, robot's speed as a function of time

$$s_t$$

- Derived, robot's acceleration as a function of time

$$a_t$$

- Derived, robot's orientation as a function of time

$$\theta_t$$

- Derived, robot's rotational velocity as a function of time

$$\omega_t$$

- Derivation

$$\frac{d^2 x}{dt^2} = a_x (t - t_0) + b_x$$

$$\frac{dx}{dt} = v_{xt} = \frac{1}{2} a_x (t - t_0)^2 + b_x (t - t_0) + K_{x1}$$

$$x(t) = x_t = \frac{1}{6} a_x (t - t_0)^3 + \frac{1}{2} b_x (t - t_0)^2 + K_{x1} (t - t_0) + K_{x2}$$

$$\frac{d^2 y}{dt^2} = a_y (t - t_0) + b_y$$

$$\frac{dy}{dt} = v_{yt} = \frac{1}{2} a_y (t - t_0)^2 + b_y (t - t_0) + K_{y1}$$

$$y(t) = y_t = \frac{1}{6} a_y (t - t_0)^3 + \frac{1}{2} b_y (t - t_0)^2 + K_{y1} (t - t_0) + K_{y2}$$

- Velocity

$$(v_{x0}, v_{y0}) = s_0 (\cos(\theta_0), \sin(\theta_0))$$

$$(v_{x1}, v_{y1}) = s_1 (\cos(\theta_1), \sin(\theta_1))$$

■ Point Start

$$v_{x0} = \frac{dx}{dt}(t_0) = S_0 \cos[\theta_0] = Dx_0 - x_0 = K_{x1}$$

$$v_{y0} = \frac{dy}{dt}(t_0) = S_0 \sin[\theta_0] = Dy_0 - y_0 = K_{y1}$$

$$x_0 = K_{x2}$$

$$y_0 = K_{y2}$$

$$K_{x1} = v_{x0}, K_{y1} = v_{y0}$$

$$K_{x2} = x_0, K_{y2} = y_0$$

$$\theta_0 = \text{ArcTan}[Dx_0 - x_0, Dy_0 - y_0] = \text{ArcTan}[v_{x0}, v_{y0}]$$

$$S_0 = \sqrt{(Dx_0 - x_0)^2 + (Dy_0 - y_0)^2}$$

$$S_0 = (Dx_0 - x_0) \cos[\theta_0] + (Dy_0 - y_0) \sin[\theta_0]$$

$$S_0 = v_{x0} \cos[\theta_0] + v_{y0} \sin[\theta_0]$$

■ Point End

$$v_{x1} = \frac{dx}{dt}(t_1) = S_1 \cos[\theta_1] = Dx_1 - x_1 = \frac{1}{2} a_x (t_1 - t_0)^2 + b_x (t_1 - t_0) + K_{x1}$$

$$v_{y1} = \frac{dy}{dt}(t_1) = S_1 \sin[\theta_1] = Dy_1 - y_1 = \frac{1}{2} a_y (t_1 - t_0)^2 + b_y (t_1 - t_0) + K_{y1}$$

$$x_1 = \frac{1}{6} a_x (t_1 - t_0)^3 + \frac{1}{2} b_x (t_1 - t_0)^2 + K_1 (t_1 - t_0) + K_{x2}$$

$$y_1 = \frac{1}{6} a_y (t_1 - t_0)^3 + \frac{1}{2} b_y (t_1 - t_0)^2 + K_{y1} (t_1 - t_0) + K_{y2}$$

$$\theta_1 = \text{ArcTan}[Dx_1 - x_1, Dy_1 - y_1] = \text{ArcTan}[v_{x1}, v_{y1}]$$

$$S_1 = \sqrt{(Dx_1 - x_1)^2 + (Dy_1 - y_1)^2}$$

$$S_1 = (Dx_1 - x_1) \cos[\theta_1] + (Dy_1 - y_1) \sin[\theta_1]$$

$$S_1 = v_{x1} \cos[\theta_1] + v_{y1} \sin[\theta_1]$$

■ Get Calculate

$$v_{x1} = \frac{1}{2} a_x (t_1 - t_0)^2 + b_x (t_1 - t_0) + v_{x0}$$

$$x_1 = \frac{1}{6} a_x (t_1 - t_0)^3 + \frac{1}{2} b_x (t_1 - t_0)^2 + v_{x0} (t_1 - t_0) + x_0$$

$$\text{Solve}\left[\left\{v_{x1} == \frac{1}{2} a_x (t_1 - t_0)^2 + b_x (t_1 - t_0) + v_{x0},\right.\right.$$

$$\left.\left.x_1 == \frac{1}{6} a_x (t_1 - t_0)^3 + \frac{1}{2} b_x (t_1 - t_0)^2 + v_{x0} (t_1 - t_0) + x_0\right\}, \{a_x, b_x\}\right] // \text{Simplify}$$

$$a_x = \frac{6 (-2 (x_0 - x_1) + (t_0 - t_1) (v_{x0} + v_{x1}))}{(t_0 - t_1)^3}$$

$$b_x = \frac{-6 (x_0 - x_1) + 2 (t_0 - t_1) (2 v_{x0} + v_{x1})}{(t_0 - t_1)^2}$$

■ Results

$$a_x = 6 \frac{((v_{x1} + v_{x0}) (t_1 - t_0) - 2 (x_1 - x_0))}{(t_1 - t_0)^3}$$

$$b_x = -2 \frac{(v_{x1} + 2 v_{x0}) (t_1 - t_0) - 3 (x_1 - x_0)}{(t_1 - t_0)^2}$$

$$a_y = 6 \frac{((v_{y1} + v_{y0}) (t_1 - t_0) - 2 (y_1 - y_0))}{(t_1 - t_0)^3}$$

$$b_y = -2 \frac{(v_{y1} + 2 v_{y0}) (t_1 - t_0) - 3 (y_1 - y_0)}{(t_1 - t_0)^2}$$

$$x_t = \frac{1}{6} a_x (t - t_0)^3 + \frac{1}{2} b_x (t - t_0)^2 + v_{x0} (t - t_0) + x_0$$

$$y_t = \frac{1}{6} a_y (t - t_0)^3 + \frac{1}{2} b_y (t - t_0)^2 + v_{y0} (t - t_0) + y_0$$

$$v_{xt} = \frac{1}{2} a_x (t - t_0)^2 + b_x (t - t_0) + v_{x0}$$

$$v_{yt} = \frac{1}{2} a_y (t - t_0)^2 + b_y (t - t_0) + v_{y0}$$

■ t0=0

$$A_x = \frac{12 (x_0 - x_1)}{T^3} + \frac{6 (v_{x0} + v_{x1})}{T^2}$$

$$B_x = -\frac{6 (x_0 - x_1)}{T^2} - \frac{2 (2 v_{x0} + v_{x1})}{T}$$

$$x_t = \frac{1}{6} A_x t^3 + \frac{1}{2} B_x t^2 + v_{x0} t + x_0$$

$$y_t = \frac{1}{6} A_y t^3 + \frac{1}{2} B_y t^2 + v_{y0} t + y_0$$

$$v_{xt} = \frac{1}{2} A_x t^2 + B_x t + v_{x0}$$

$$v_{yt} = \frac{1}{2} A_y t^2 + B_y t + v_{y0}$$

■ orientation

$$\theta_t = \text{ArcTan}[v_{xt}, v_{yt}]$$

■ speed

$$S_t = v_{xt} \cos[\theta_t] + v_{yt} \sin[\theta_t]$$

■ x, y velocity

$$v_{xt} = S_t \cos[\theta_t]$$

$$v_{yt} = S_t \sin[\theta_t]$$

■ arc length

$$L = \int_{t_0}^{t_1} S_t dt = \int_{t_0}^{t_1} \sqrt{v_{xt}^2 + v_{yt}^2} dt$$

■ robot speed

$$s_t = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

$$s_t = \sqrt{\left(\left(\frac{1}{2} a_x (t - t_0)^2 + b_x (t - t_0) + v_{x0}\right)^2 + \left(\frac{1}{2} a_y (t - t_0)^2 + b_y (t - t_0) + v_{y0}\right)^2\right)}$$

$$s_t = \sqrt{\left(\left(\frac{1}{2} a_x t^2 + b_x t + v_{x0}\right)^2 + \left(\frac{1}{2} a_y t^2 + b_y t + v_{y0}\right)^2\right)}$$

$$s_t = \sqrt{\left(\frac{(a_x^2 + a_y^2)}{4} t^4 + (a_x b_x + a_y b_y) t^3 + (b_x^2 + b_y^2 + a_x v_{x0} + a_y v_{y0}) t^2 + 2 (b_x v_{x0} + b_y v_{y0}) t + (v_{x0}^2 + v_{y0}^2)\right)}$$

$$\begin{pmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{pmatrix} = \begin{pmatrix} v_{x0}^2 + v_{y0}^2 \\ 2 (b_x v_{x0} + b_y v_{y0}) \\ b_x^2 + b_y^2 + a_x v_{x0} + a_y v_{y0} \\ a_x b_x + a_y b_y \\ \frac{1}{4} (a_x^2 + a_y^2) \end{pmatrix}$$

$$(\because t_0 = 0)$$

$$s_t = \sqrt{p_4 t^4 + p_3 t^3 + p_2 t^2 + p_1 t + p_0}$$

■ Length

$$L = \int_{t_0}^{t_1} S_t dt = \int_{t_0}^{t_1} \sqrt{v_{xt}^2 + v_{yt}^2} dt$$

$$\int_0^T \sqrt{\left(\frac{1}{2} A_x t^2 + B_x t + v_{x0}\right)^2 + \left(\frac{1}{2} A_y t^2 + B_y t + v_{y0}\right)^2} dt$$

$$\int_0^T \sqrt{\left(\frac{(A_x^2 + A_y^2)}{4} t^4 + (A_x B_x + A_y B_y) t^3 + (B_x^2 + B_y^2 + A_x v_{x0} + A_y v_{y0}) t^2 + 2 (B_x v_{x0} + B_y v_{y0}) t + (v_{x0}^2 + v_{y0}^2)\right)} dt$$

$$\int_0^T \sqrt{P_4 t^4 + P_3 t^3 + P_2 t^2 + P_1 t + P_0} dt$$

■ acceleration

$$a_t = \frac{d^2 s}{dt^2} = \partial_t \sqrt{p_4 t^4 + p_3 t^3 + p_2 t^2 + p_1 t + p_0} = \frac{p_1 + 2 t p_2 + 3 t^2 p_3 + 4 t^3 p_4}{2 \sqrt{p_0 + t p_1 + t^2 p_2 + t^3 p_3 + t^4 p_4}}$$

■ orientation

$$\theta_t = \text{ArcTan}\left[\frac{v_{yt}}{v_{xt}}\right] = \text{ArcTan}\left[\frac{\frac{1}{2} a_y t^2 + b_y t + v_{y0}}{\frac{1}{2} a_x t^2 + b_x t + v_{x0}}\right]$$

■ rotational velocity

$$\begin{aligned} \omega_t &= \frac{d\theta}{dt} = \partial_t \text{ArcTan}\left[\frac{\frac{1}{2} a_y t^2 + b_y t + v_{y0}}{\frac{1}{2} a_x t^2 + b_x t + v_{x0}}\right] \\ \omega_t &= \frac{(a_y t + b_y) \left(\frac{1}{2} a_x t^2 + b_x t + v_{x0}\right) - (a_x t + b_x) \left(\frac{1}{2} a_y t^2 + b_y t + v_{y0}\right)}{\left(\frac{1}{2} a_x t^2 + b_x t + v_{x0}\right)^2 + \left(\frac{1}{2} a_y t^2 + b_y t + v_{y0}\right)^2} \\ \omega_t &= \frac{(a_y t + b_y) v_{xt} - (a_x t + b_x) v_{yt}}{v_{xt}^2 + v_{yt}^2} \end{aligned}$$

Robot 2 Wheel

$$\text{LinearVelocity} = \frac{\text{LeftSpeed} + \text{RightSpeed}}{2}$$

$$\text{AngularVelocity} = \frac{\text{RightSpeed} - \text{LeftSpeed}}{\text{WheelBase}}$$

$$\text{LeftSpeed} = \text{LinearVelocity} - \frac{\text{WheelBase}}{2} * \text{AngularVelocity}$$

$$\text{RightSpeed} = \text{LinearVelocity} + \frac{\text{WheelBase}}{2} * \text{AngularVelocity}$$

■ left wheel position

$$\left\{ x_t + \frac{\text{WheelBase}}{2} * \cos\left[\theta_t + \frac{\pi}{2}\right], y_t + \frac{\text{WheelBase}}{2} * \sin\left[\theta_t + \frac{\pi}{2}\right] \right\}$$

■ right wheel position

$$\left\{ x_t + \frac{\text{WheelBase}}{2} * \cos\left[\theta_t - \frac{\pi}{2}\right], y_t + \frac{\text{WheelBase}}{2} * \sin\left[\theta_t - \frac{\pi}{2}\right] \right\}$$

■ left wheel speed

$$\text{Speed}_t = \frac{\text{WheelBase}}{2} * \omega_t$$

■ right wheel speed

$$\text{Speed}_t + \frac{\text{WheelBase}}{2} * \omega_t$$

Vector

■ waypoints

$$\mathbf{p}_{i-1}, \mathbf{p}_i, \mathbf{p}_{i+1}$$

$$\mathbf{a}_i = \mathbf{p}_i - \mathbf{p}_{i-1}$$

$$\mathbf{b}_i = \mathbf{p}_{i+1} - \mathbf{p}_i$$

$$\mathbf{u}_i = \frac{\mathbf{a}_i \|\mathbf{b}_i\| + \mathbf{b}_i \|\mathbf{a}_i\|}{\|\mathbf{a}_i\| \|\mathbf{b}_i\| + \mathbf{b}_i \|\mathbf{a}_i\|} = \frac{\mathbf{a}_i \|\mathbf{b}_i\| + \mathbf{b}_i \|\mathbf{a}_i\|}{2 \|\mathbf{a}_i\| \|\mathbf{b}_i\| \sqrt{\frac{1}{2} \left(1 + \frac{\mathbf{a}_i \cdot \mathbf{b}_i}{\|\mathbf{a}_i\| \|\mathbf{b}_i\|}\right)}}$$

$$\mathbf{p} \cdot \mathbf{u} = 0$$

$$\cos\left(\frac{\pi - \theta}{2}\right) = \frac{\mathbf{p} \cdot \mathbf{b}}{\|\mathbf{p}\| \|\mathbf{b}\|} = \frac{\mathbf{p} \cdot (-\mathbf{a})}{\|\mathbf{p}\| \|\mathbf{a}\|}$$

$$\frac{\mathbf{p} \cdot \mathbf{b}}{\|\mathbf{p}\| \|\mathbf{b}\|} - \frac{\mathbf{p} \cdot (-\mathbf{a})}{\|\mathbf{p}\| \|\mathbf{a}\|} = 0$$

$$\frac{\mathbf{p} \cdot \mathbf{b}}{\|\mathbf{p}\| \|\mathbf{b}\|} + \frac{\mathbf{p} \cdot \mathbf{a}}{\|\mathbf{p}\| \|\mathbf{a}\|} = 0$$

$$\frac{\mathbf{p} \cdot (\mathbf{b} \|\mathbf{a}\|)}{1} + \frac{\mathbf{p} \cdot (\mathbf{a} \|\mathbf{b}\|)}{1} = 0 * \|\mathbf{p}\| \|\mathbf{a}\| \|\mathbf{b}\|$$

$$\mathbf{p} \cdot (\mathbf{b} \|\mathbf{a}\| + \mathbf{a} \|\mathbf{b}\|) = 0$$

$$\therefore \mathbf{p} \cdot \mathbf{u} = 0$$

$$\mathbf{p} \cdot (\mathbf{b} \|\mathbf{a}\| + \mathbf{a} \|\mathbf{b}\|) = 0$$

$$\therefore \mathbf{u} = \mathbf{a} \|\mathbf{b}\| + \mathbf{b} \|\mathbf{a}\|$$

$$\cos(\theta_i) = \frac{\mathbf{a}_i \cdot \mathbf{b}_i}{\|\mathbf{a}_i\| \|\mathbf{b}_i\|}$$

$$\mathbf{a}_i \cdot (\mathbf{a}_{iy}, -\mathbf{a}_{ix}) = \mathbf{a}_{ix} \mathbf{a}_{iy} - \mathbf{a}_{iy} \mathbf{a}_{ix} = 0$$

$$\mathbf{m}_i = \frac{(\mathbf{a}_{iy}, -\mathbf{a}_{ix})}{\|\mathbf{a}_i\|}$$

$$\sin(\theta_{u\omega}) = \frac{\mathbf{u}_x \omega_y - \mathbf{u}_y \omega_x}{\|\mathbf{u}\| \|\hat{\omega}\|}$$

■ Trajectory

$$\phi_{i,1} = \text{ArcCos} \left(\frac{\vec{a}_{i+1} \cdot \vec{v}_i}{|\vec{a}_{i+1}| |\vec{v}_i|} \right)$$

$$L_{i,1} = \left| \frac{|\vec{a}_{i+1}| \phi_{i,1}}{\text{Sin}(\phi_{i,1})} \right|$$

$$\phi_{i,2} = \text{ArcCos} \left(\frac{\vec{a}_{i+1} \cdot \vec{v}_{i+1}}{|\vec{a}_{i+1}| |\vec{v}_{i+1}|} \right)$$

$$L_{i,2} = \left| \frac{|\vec{a}_{i+1}| \phi_{i,2}}{\text{Sin}(\phi_{i,2})} \right|$$

$$\Delta r = \frac{L_{i,1} + L_{i,2}}{s_i + s_{i+1}}$$

Circular Arc

$$x = r \text{Cos}(\phi)$$

$$y = r \text{Sin}(\phi)$$

$$r = \sqrt{x^2 + y^2}$$

$$\phi = \text{ArcTan} \left(\frac{y}{x} \right)$$

$$\text{Sin}(a \pm b) = \text{Sin}(a) \text{Cos}(b) \pm \text{Cos}(a) \text{Sin}(b)$$

$$\text{Sin}(2a) = 2 \text{Sin}(a) \text{Cos}(a)$$

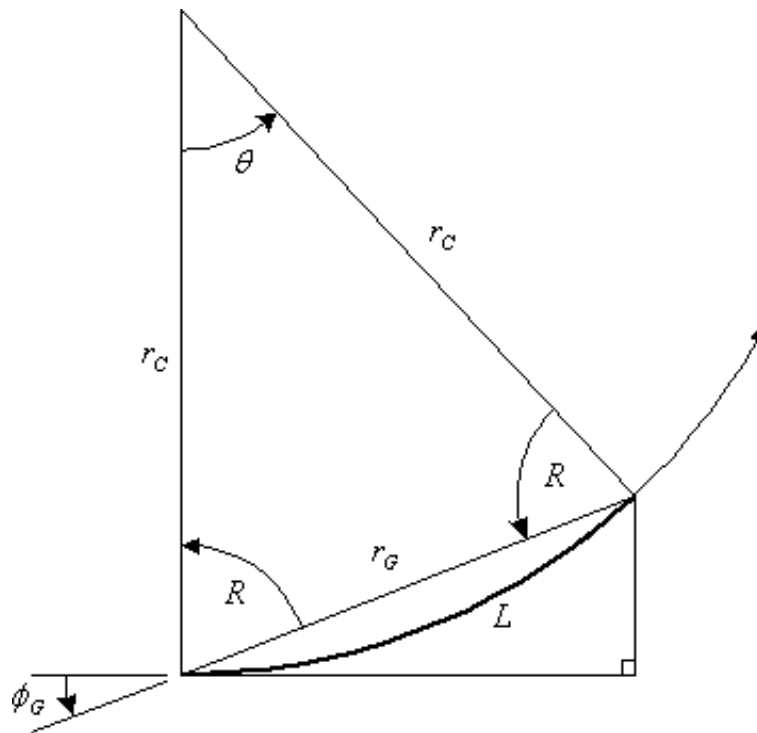
$$\text{Sin} \left(a + \frac{\pi}{2} \right) = \text{Cos}(a)$$

$$\text{Sin} \left(a - \frac{\pi}{2} \right) = -\text{Cos}(a)$$

$$\text{Cos} \left(a + \frac{\pi}{2} \right) = -\text{Sin}(a)$$

$$\text{Cos} \left(a - \frac{\pi}{2} \right) = \text{Sin}(a)$$

■ Trajectory based on an arc of a circle



$$\frac{\sin(\theta)}{r_g} = \frac{\sin(R)}{r_c}$$

$$\phi_G + R = \frac{\pi}{2}$$

$$\theta + 2R = \pi$$

$$\theta + 2\left(\frac{\pi}{2} - \phi_G\right) = \pi$$

$$\theta = 2\phi_G$$

$$\frac{\sin(2\phi_G)}{r_g} = \frac{\sin\left(\frac{\pi}{2} - \phi_G\right)}{r_c}$$

$$r_c = r_g \frac{\sin\left(\frac{\pi}{2} - \phi_G\right)}{\sin(2\phi_G)}$$

$$r_c = r_g \frac{\cos(\phi_G)}{2\sin(\phi_G)\cos(\phi_G)}$$

$$r_c = \frac{r_g}{2\sin(\phi_G)}$$

$$L = r_c \theta$$

$$L = r_c (2\phi_G)$$

$$L = \frac{r_g \cdot \phi_G}{\sin(\phi_G)}$$

Geometry in Motion

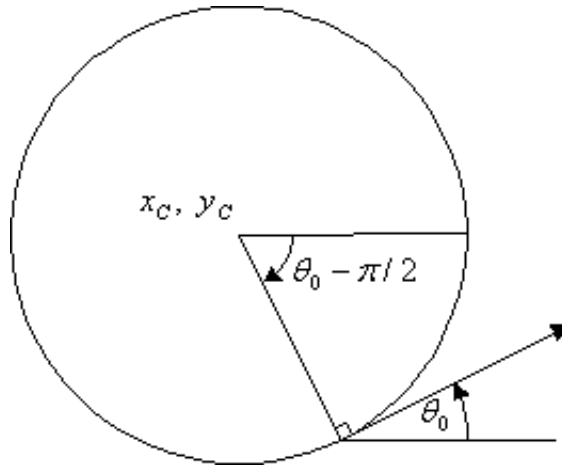
$$t_1 - t_0 = \frac{L}{s}$$

$$\omega = \frac{\theta}{t_1 - t_0}$$

$$\theta(t) = \omega(t - t_0) + \theta_0$$

Trigonometry at Work

■ Relation between initial point and circle.



$$x_c = x_0 + r_c \cdot \cos\left(\theta_0 + \frac{\pi}{2}\right)$$

$$y_c = y_0 + r_c \cdot \sin\left(\theta_0 + \frac{\pi}{2}\right)$$

$$x_c = x_0 - |r_c| \cos\left(\theta_0 - \frac{\pi}{2}\right)$$

$$y_c = y_0 - |r_c| \sin\left(\theta_0 - \frac{\pi}{2}\right)$$

$$x_c = x_0 - r_c \cdot \sin(\theta_0)$$

$$y_c = y_0 + r_c \cdot \cos(\theta_0)$$

$$x(t) = x_c + r_c \cdot \cos\left(\omega(t - t_0) + \theta_0 - \frac{\pi}{2}\right)$$

$$y(t) = y_0 + r_c \cdot \sin\left(\omega(t - t_0) + \theta_0 - \frac{\pi}{2}\right)$$

$$x(t) = x_c - r_c \cdot \sin(\theta_0) + r_c \cdot \sin(\omega(t - t_0) + \theta_0)$$

$$y(t) = y_0 + r_c \cdot \cos(\theta_0) - r_c \cdot \cos(\omega(t - t_0) + \theta_0)$$

■ Results

$$\text{if } \phi_6 > \frac{\pi}{2} \text{ then } \theta = 2(\phi_6 - \pi)$$

$$\text{if } \phi_6 < -\frac{\pi}{2} \text{ then } \theta = 2(\phi_6 + \pi)$$

$$L = -r_c \cdot \theta$$

$$t_1 - t_0 = \frac{L}{s}$$

$$\omega = \frac{\theta}{t_1 - t_0}$$