

Curve

■ Derivation

$$\frac{d^2x}{dt^2} = a_x (t - t_0) + b_x$$

$$\frac{dx}{dt} = v_{xt} = \frac{1}{2} a_x (t - t_0)^2 + b_x (t - t_0) + K_{x1}$$

$$x(t) = x_t = \frac{1}{6} a_x (t - t_0)^3 + \frac{1}{2} b_x (t - t_0)^2 + K_{x1} (t - t_0) + K_{x2}$$

$$\frac{d^2y}{dt^2} = a_y (t - t_0) + b_y$$

$$\frac{dy}{dt} = v_{yt} = \frac{1}{2} a_y (t - t_0)^2 + b_y (t - t_0) + K_{y1}$$

$$y(t) = y_t = \frac{1}{6} a_y (t - t_0)^3 + \frac{1}{2} b_y (t - t_0)^2 + K_{y1} (t - t_0) + K_{y2}$$

■ Point Start

$$v_{x0} = \frac{dx}{dt}(t_0) = S_0 \cos[\theta_0] = Dx_0 - x_0 = K_{x1}$$

$$v_{y0} = \frac{dy}{dt}(t_0) = S_0 \sin[\theta_0] = Dy_0 - y_0 = K_{y1}$$

$$x_0 = K_{x2}$$

$$y_0 = K_{y2}$$

$$K_{x1} = v_{x0}, \quad K_{y1} = v_{y0}$$

$$K_{x2} = x_0, \quad K_{y2} = y_0$$

$$\theta_0 = \text{ArcTan}[Dx_0 - x_0, Dy_0 - y_0] = \text{ArcTan}[v_{x0}, v_{y0}]$$

$$S_0 = \sqrt{(Dx_0 - x_0)^2 + (Dy_0 - y_0)^2}$$

$$S_0 = (Dx_0 - x_0) \cos[\theta_0] + (Dy_0 - y_0) \sin[\theta_0]$$

$$S_0 = v_{x0} \cos[\theta_0] + v_{y0} \sin[\theta_0]$$

■ Point End

$$v_{x1} = \frac{dx}{dt}(t_1) = S_1 \cos[\theta_1] = Dx_1 - x_1 = \frac{1}{2} a_x (t_1 - t_0)^2 + b_x (t_1 - t_0) + K_{x1}$$

$$v_{y1} = \frac{dy}{dt}(t_1) = S_1 \sin[\theta_1] = Dy_1 - y_1 = \frac{1}{2} a_y (t_1 - t_0)^2 + b_y (t_1 - t_0) + K_{y1}$$

$$x_1 = \frac{1}{6} a_x (t_1 - t_0)^3 + \frac{1}{2} b_x (t_1 - t_0)^2 + K_{x1} (t_1 - t_0) + K_{x2}$$

$$y_1 = \frac{1}{6} a_y (t_1 - t_0)^3 + \frac{1}{2} b_y (t_1 - t_0)^2 + K_{y1} (t_1 - t_0) + K_{y2}$$

$$\theta_1 = \text{ArcTan}[Dx_1 - x_1, Dy_1 - y_1] = \text{ArcTan}[v_{x1}, v_{y1}]$$

$$S_1 = \sqrt{(Dx_1 - x_1)^2 + (Dy_1 - y_1)^2}$$

$$S_1 = (Dx_1 - x_1) \cos[\theta_1] + (Dy_1 - y_1) \sin[\theta_1]$$

$$S_1 = v_{x1} \cos[\theta_1] + v_{y1} \sin[\theta_1]$$

■ Results

$$\text{Solve}\left[\left\{v_{x1} = \frac{1}{2} a_x (t_1 - t_0)^2 + b_x (t_1 - t_0) + v_{x0},\right.\right.$$

$$x_1 = \frac{1}{6} a_x (t_1 - t_0)^3 + \frac{1}{2} b_x (t_1 - t_0)^2 + v_{x0} (t_1 - t_0) + x_0\left.\right\}, \{a_x, b_x\}]$$

$$a_x = \frac{6(-2(x_0 - x_1) + (t_0 - t_1)(v_{x0} + v_{x1}))}{(t_0 - t_1)^3}$$

$$b_x = \frac{2(-3(x_0 - x_1) + (t_0 - t_1)(2v_{x0} + v_{x1}))}{(t_0 - t_1)^2}$$

$$a_y = \frac{6(-2(y_0 - y_1) + (t_0 - t_1)(v_{y0} + v_{y1}))}{(t_0 - t_1)^3}$$

$$b_y = \frac{2(-3(y_0 - y_1) + (t_0 - t_1)(2v_{y0} + v_{y1}))}{(t_0 - t_1)^2}$$

■ orientation

$$\theta_t = \text{ArcTan}[v_{xt}, v_{yt}]$$

■ speed

$$S_t = v_{xt} \cos[\theta_t] + v_{yt} \sin[\theta_t]$$

■ x, y velocity

$$v_{xt} = S_t \cos[\theta_t]$$

$$v_{yt} = S_t \sin[\theta_t]$$

■ arc length

$$L = \int_{t_0}^{t_1} S_t dt = \int_{t_0}^{t_1} \sqrt{v_{xt}^2 + v_{yt}^2} dt$$

Data

■ Acceleration

$$\frac{d^2 x}{dt^2} = a_x (t - t_0) + b_x$$

$$\frac{d^2 y}{dt^2} = a_y (t - t_0) + b_y$$

■ velocity

$$\begin{aligned}\frac{dx}{dt} &= \frac{1}{2} a_x (t - t_0)^2 + b_x (t - t_0) + K_{x1} = \frac{1}{2} a_x (t - t_0)^2 + b_x (t - t_0) + v_{x0} \\ &= \left(\frac{1}{(t_0 - t_1)^3} (-6 (t - t_0) (t - t_1) (x_0 - x_1) + \right. \\ &\quad \left. (t_0 - t_1) (-t + t_1) (-3t + 2t_0 + t_1) v_{x0} - (-t + t_0) (-t_0 + t_1) (-3t + t_0 + 2t_1) v_{x1}) \right) \\ \frac{dy}{dt} &= \frac{1}{2} a_y (t - t_0)^2 + b_y (t - t_0) + K_{y1} = \frac{1}{2} a_y (t - t_0)^2 + b_y (t - t_0) + v_{y0} \\ &= \left(\frac{1}{(t_0 - t_1)^3} (-6 (t - t_0) (t - t_1) (y_0 - y_1) + \right. \\ &\quad \left. (t_0 - t_1) (-t + t_1) (-3t + 2t_0 + t_1) v_{y0} - (-t + t_0) (-t_0 + t_1) (-3t + t_0 + 2t_1) v_{y1}) \right)\end{aligned}$$

■ point

$$\begin{aligned}x(t) &= \frac{1}{6} a_x (t - t_0)^3 + \frac{1}{2} b_x (t - t_0)^2 + K_1 (t - t_0) + K_{x2} \\ &= \left(\frac{1}{(t_0 - t_1)^3} (- (t - t_1)^2 (2t - 3t_0 + t_1) x_0 + \right. \\ &\quad \left. (t - t_0) ((t - t_0) (2t + t_0 - 3t_1) x_1 + (t - t_1) (t_0 - t_1) ((t - t_1) v_{x0} + (t - t_0) v_{x1})) \right) \\ y(t) &= \frac{1}{6} a_y (t - t_0)^3 + \frac{1}{2} b_y (t - t_0)^2 + K_{y1} (t - t_0) + K_{y2} \\ &= \left(\frac{1}{(t_0 - t_1)^3} (- (t - t_1)^2 (2t - 3t_0 + t_1) y_0 + \right. \\ &\quad \left. (t - t_0) ((t - t_0) (2t + t_0 - 3t_1) y_1 + (t - t_1) (t_0 - t_1) ((t - t_1) v_{y0} + (t - t_0) v_{y1})) \right)\end{aligned}$$

■ $t_0 = 0$

■ velocity

$$\begin{aligned}v_{xt} &= \frac{6t^2 x_0}{t_1^3} - \frac{6t x_0}{t_1^2} - \frac{6t^2 x_1}{t_1^3} + \frac{6t x_1}{t_1^2} + v_{x0} + \frac{3t^2 v_{x0}}{t_1^2} - \frac{4t v_{x0}}{t_1} + \frac{3t^2 v_{x1}}{t_1^2} - \frac{2t v_{x1}}{t_1} \\ v_{yt} &= \frac{6t^2 y_0}{t_1^3} - \frac{6t y_0}{t_1^2} - \frac{6t^2 y_1}{t_1^3} + \frac{6t y_1}{t_1^2} + v_{y0} + \frac{3t^2 v_{y0}}{t_1^2} - \frac{4t v_{y0}}{t_1} + \frac{3t^2 v_{y1}}{t_1^2} - \frac{2t v_{y1}}{t_1}\end{aligned}$$

■ x_0, x_1, v_{x0}, v_{x1}

$$\begin{aligned}v_{xt} &= \frac{6t(t-t_1)x_0}{t_1^3} - \frac{6t(t-t_1)x_1}{t_1^3} + \frac{(t-t_1)(3t-t_1)v_{x0}}{t_1^2} + \frac{t(3t-2t_1)v_{x1}}{t_1^2} \\ v_{yt} &= \frac{6t(t-t_1)y_0}{t_1^3} - \frac{6t(t-t_1)y_1}{t_1^3} + \frac{(t-t_1)(3t-t_1)v_{y0}}{t_1^2} + \frac{t(3t-2t_1)v_{y1}}{t_1^2}\end{aligned}$$

■ t

$$v_{xt} = \frac{3 t^2 (2 x_0 - 2 x_1 + t_1 v_{x0} + t_1 v_{x1})}{t_1^3} - \frac{2 t (3 x_0 - 3 x_1 + 2 t_1 v_{x0} + t_1 v_{x1})}{t_1^2} + v_{x0}$$

$$v_{yt} = \frac{3 t^2 (2 y_0 - 2 y_1 + t_1 v_{y0} + t_1 v_{y1})}{t_1^3} - \frac{2 t (3 y_0 - 3 y_1 + 2 t_1 v_{y0} + t_1 v_{y1})}{t_1^2} + v_{y0}$$

■ t_1

$$v_{xt} = \frac{6 t^2 (x_0 - x_1)}{t_1^3} + \frac{3 t (-2 x_0 + 2 x_1 + t v_{x0} + t v_{x1})}{t_1^2} - \frac{2 t (2 v_{x0} + v_{x1})}{t_1} + v_{x0}$$

$$v_{yt} = \frac{6 t^2 (y_0 - y_1)}{t_1^3} + \frac{3 t (-2 y_0 + 2 y_1 + t v_{y0} + t v_{y1})}{t_1^2} - \frac{2 t (2 v_{y0} + v_{y1})}{t_1} + v_{y0}$$

■ point

$$x_t = x_0 + \frac{2 t^3 x_0}{t_1^3} - \frac{3 t^2 x_0}{t_1^2} - \frac{2 t^3 x_1}{t_1^3} + \frac{3 t^2 x_1}{t_1^2} + t v_{x0} + \frac{t^3 v_{x0}}{t_1^2} - \frac{2 t^2 v_{x0}}{t_1} + \frac{t^3 v_{x1}}{t_1^2} - \frac{t^2 v_{x1}}{t_1}$$

$$y_t = y_0 + \frac{2 t^3 y_0}{t_1^3} - \frac{3 t^2 y_0}{t_1^2} - \frac{2 t^3 y_1}{t_1^3} + \frac{3 t^2 y_1}{t_1^2} + t v_{y0} + \frac{t^3 v_{y0}}{t_1^2} - \frac{2 t^2 v_{y0}}{t_1} + \frac{t^3 v_{y1}}{t_1^2} - \frac{t^2 v_{y1}}{t_1}$$

■ x_0, x_1, v_{x0}, v_{x1}

$$x_t = \frac{(t - t_1)^2 (2 t + t_1) x_0}{t_1^3} - \frac{t^2 (2 t - 3 t_1) x_1}{t_1^3} + \frac{t (t - t_1)^2 v_{x0}}{t_1^2} + \frac{t^2 (t - t_1) v_{x1}}{t_1^2}$$

$$y_t = \frac{(t - t_1)^2 (2 t + t_1) y_0}{t_1^3} - \frac{t^2 (2 t - 3 t_1) y_1}{t_1^3} + \frac{t (t - t_1)^2 v_{y0}}{t_1^2} + \frac{t^2 (t - t_1) v_{y1}}{t_1^2}$$

■ t

$$x_t = \frac{(2 (x_0 - x_1) + t_1 (v_{x0} + v_{x1}))}{t_1^3} t^3 - \frac{(3 (x_0 - x_1) + t_1 (2 v_{x0} + v_{x1}))}{t_1^2} t^2 + v_{x0} t + x_0$$

$$y_t = \frac{(2 (y_0 - y_1) + t_1 (v_{y0} + v_{y1}))}{t_1^3} t^3 - \frac{(3 (y_0 - y_1) + t_1 (2 v_{y0} + v_{y1}))}{t_1^2} t^2 + v_{y0} t + y_0$$

■ t_1

$$x_t = \frac{2 t^3 (x_0 - x_1)}{t_1^3} + \frac{t^2 (-3 x_0 + 3 x_1 + t v_{x0} + t v_{x1})}{t_1^2} - \frac{t^2 (2 v_{x0} + v_{x1})}{t_1} + x_0 + t v_{x0}$$

$$y_t = \frac{2 t^3 (y_0 - y_1)}{t_1^3} + \frac{t^2 (-3 y_0 + 3 y_1 + t v_{y0} + t v_{y1})}{t_1^2} - \frac{t^2 (2 v_{y0} + v_{y1})}{t_1} + t v_{y0} + y_0$$

Point

$$x_t = \frac{(2(x_0 - x_1) + t_1(v_{x0} + v_{x1}))}{t_1^3} t^3 - \frac{(3(x_0 - x_1) + t_1(2v_{x0} + v_{x1}))}{t_1^2} t^2 + v_{x0} t + x_0$$

$$y_t = \frac{(2(y_0 - y_1) + t_1(v_{y0} + v_{y1}))}{t_1^3} t^3 - \frac{(3(y_0 - y_1) + t_1(2v_{y0} + v_{y1}))}{t_1^2} t^2 + v_{y0} t + y_0$$

$$v_{xt} = \frac{3t^2(2x_0 - 2x_1 + t_1v_{x0} + t_1v_{x1})}{t_1^3} - \frac{2t(3x_0 - 3x_1 + 2t_1v_{x0} + t_1v_{x1})}{t_1^2} + v_{x0}$$

$$v_{yt} = \frac{3t^2(2y_0 - 2y_1 + t_1v_{y0} + t_1v_{y1})}{t_1^3} - \frac{2t(3y_0 - 3y_1 + 2t_1v_{y0} + t_1v_{y1})}{t_1^2} + v_{y0}$$

$$x_t = \left(\frac{2(x_0 - x_1)}{t_1^3} + \frac{v_{x0} + v_{x1}}{t_1^2} \right) t^3 + \left(-\frac{3(x_0 - x_1)}{t_1^2} - \frac{2v_{x0} + v_{x1}}{t_1} \right) t^2 + v_{x0} t + x_0$$

$$y_t = \left(\frac{2(y_0 - y_1)}{t_1^3} + \frac{v_{y0} + v_{y1}}{t_1^2} \right) t^3 + \left(-\frac{3(y_0 - y_1)}{t_1^2} - \frac{2v_{y0} + v_{y1}}{t_1} \right) t^2 + v_{y0} t + y_0$$

■ ∴

$$x_t = \frac{1}{6} A_x t^3 + \frac{1}{2} B_x t^2 + v_{x0} t + x_0$$

$$y_t = \frac{1}{6} A_y t^3 + \frac{1}{2} B_y t^2 + v_{y0} t + y_0$$

$$v_{xt} = \frac{1}{2} A_x t^2 + B_x t + v_{x0}$$

$$v_{yt} = \frac{1}{2} A_y t^2 + B_y t + v_{y0}$$

$$A_x = \frac{12(x_0 - x_1)}{T^3} + \frac{6(v_{x0} + v_{x1})}{T^2}$$

$$B_x = -\frac{6(x_0 - x_1)}{T^2} - \frac{2(2v_{x0} + v_{x1})}{T}$$

■ Length

$$L = \int_{t_0}^{t_1} S_t dt = \int_{t_0}^{t_1} \sqrt{v_{xt}^2 + v_{yt}^2} dt$$

$$\int_0^T \sqrt{\left(\frac{1}{2} A_x t^2 + B_x t + v_{x0} \right)^2 + \left(\frac{1}{2} A_y t^2 + B_y t + v_{y0} \right)^2} dt$$

$$\int_0^T$$

$$\sqrt{\frac{(A_x^2 + A_y^2)}{4} t^4 + (A_x B_x + A_y B_y) t^3 + (B_x^2 + B_y^2 + A_x v_{x0} + A_y v_{y0}) t^2 + 2(B_x v_{x0} + B_y v_{y0}) t + (v_{x0}^2 + v_{y0}^2)} dt$$

$$\int_0^T \sqrt{\frac{(A_x^2 + A_y^2)}{4} t^4 + (A_x B_x + A_y B_y) t^3 + (B_x^2 + B_y^2 + A_x v_{x0} + A_y v_{y0}) t^2 + 2 (B_x v_{x0} + B_y v_{y0}) t + (v_{x0}^2 + v_{y0}^2)} dt$$

$$\int_0^T \sqrt{P_4 t^4 + P_3 t^3 + P_2 t^2 + P_1 t + P_0} dt$$

T,t

$$x[T, t] = \frac{1}{6} A_x[T] t^3 + \frac{1}{2} B_x[T] t^2 + v_{x0} t + x_0$$

$$y[T, t] = \frac{1}{6} A_y[T] t^3 + \frac{1}{2} B_y[T] t^2 + v_{y0} t + y_0$$

$$v_x[T, t] = \frac{1}{2} A_x[T] t^2 + B_x[T] t + v_{x0}$$

$$v_y[T, t] = \frac{1}{2} A_y[T] t^2 + B_y[T] t + v_{y0}$$

$$A_x[T] = \frac{12 (x_0 - x_1)}{T^3} + \frac{6 (v_{x0} + v_{x1})}{T^2}$$

$$B_x[T] = -\frac{6 (x_0 - x_1)}{T^2} - \frac{2 (2 v_{x0} + v_{x1})}{T}$$

∴ T, S_t, L

T를 늘리면 S_t가 낮아지고 L 거리는 짧아지고
T가 짧으면 S_t는 높아지고 L 거리는 길어진다.

S_t 범위를 제한 하면 T, L이 결정된다