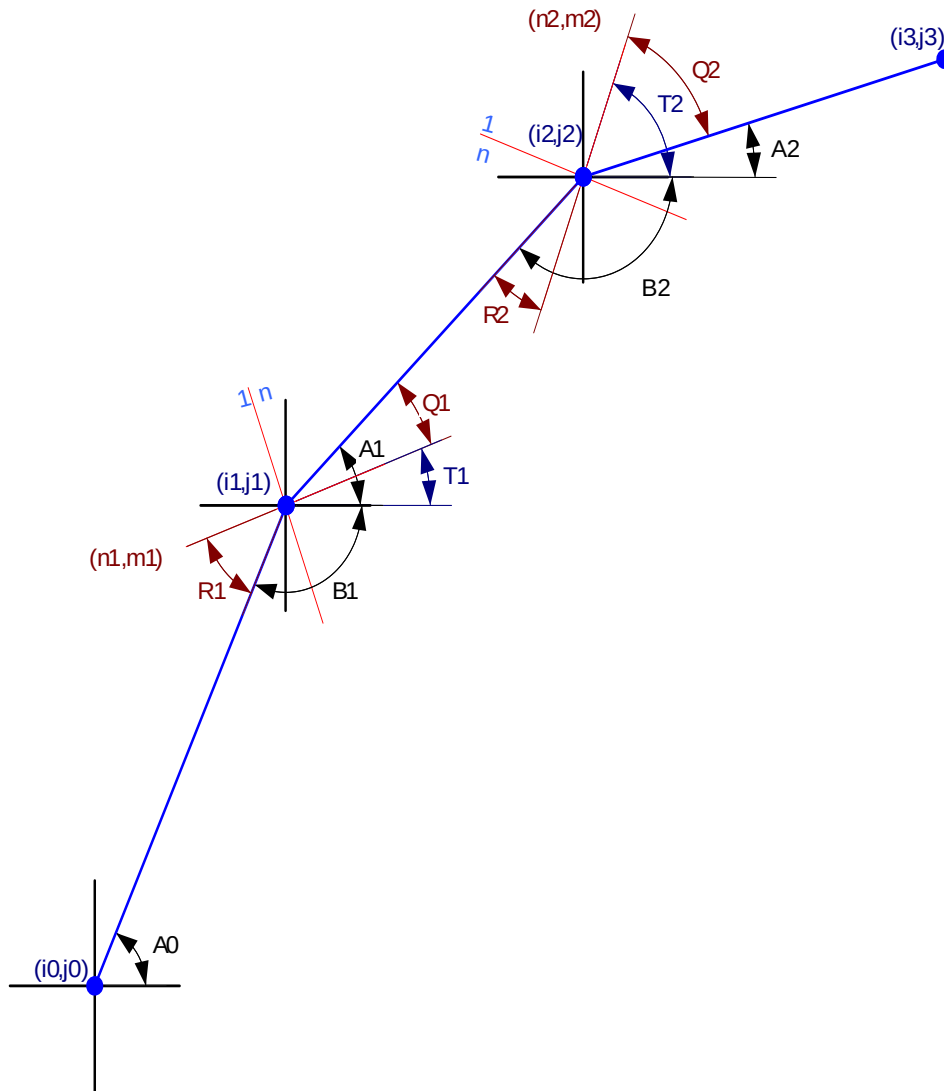


Refraction



$$A0 = \text{ArcTan}[i1 - i0, j1 - j0]$$

$$A1 = \text{ArcTan}[i2 - i1, j2 - j1]$$

$$T1 = \text{ArcTan}[n1 - i1, m1 - j1]$$

$$B1 = A0 - \pi$$

$$n \sin[Q1] = \sin[R1]$$

$$n \sin[T1 - A1] = \sin[B1 - T1]$$

$$n \sin[T1 - A1] = -\sin[A0 - T1]$$

$$A1 \rightarrow T1 + \text{ArcSin}[\sin[A0 - T1] / n]$$

$$A2 = \text{ArcTan}[i3 - i2, j3 - j2]$$

$$T2 = \text{ArcTan}[n2 - i2, m2 - j2]$$

$$B2 = A1 - \pi$$

$$\sin[Q2] = n \sin[R2]$$

$$\sin[T2 - A2] = n \sin[B2 - T2]$$

$$\sin[T2 - A2] = -n \sin[A1 - T2]$$

$$A2 \rightarrow T2 + \text{ArcSin}[n \sin[A1 - T2]]$$

Surface1

$$y == a1 x^2 + b1$$

$$\theta == a1 x^2 - (y - b1)$$

■ Point1

$$P1 : \{I1, J1, \theta\}$$

■ V1

$$\{2 a1 I1, -1, \theta\}$$

■ L1

$$\frac{x - I1}{2 a1 I1} == \frac{y - J1}{-1} == t1$$

■ eP1

$$\{2 a1 I1 t1 + I1, J1 - t1, \theta\}$$

$$a1 (2 a1 I1 t1 + I1)^2 - (J1 - t1 - b1) == 0$$

$$a1 I1^2 + b1 == J1$$

$$\{I1, a1 I1^2 + b1, \theta\}$$

$$\{n1, m1, \theta\} == \{-2 a1 I1 + I1, J1 - 1, \theta\}$$

■ A0

$$A0 = \text{ArcTan}[I1 - I0, J1 - J0]$$

$$A1 = \text{ArcTan}[I2 - I1, J2 - J1]$$

$$T1 = \text{ArcTan}[n1 - I1, m1 - J1]$$

$$B1 = A0 - \pi$$

$$n \sin[Q1] = \sin[R1]$$

$$n \sin[T1 - A1] == \sin[B1 - T1]$$

$$n \sin[T1 - A1] == -\sin[A0 - T1]$$

$$\text{Solve}[n \sin[T1 - A1] == -\sin[A0 - T1], A1] // \text{InputForm}$$

$$A1 \rightarrow T1 + \text{ArcSin}[\sin[A0 - T1] / n]$$

Surface2

$$y == b2$$

$$\theta == -(y - b2)$$

■ Point2

$$P2 : \{I2, J2, 0\}$$

$$\text{Solve}[\{J2 == \tan[A1] (I2 - I1) + J1, J2 == b2\}, \{I2, J2\}]$$

$$\{\{I2 \rightarrow -\csc[A1] (-b2 \cos[A1] + J1 \cos[A1] - I1 \sin[A1]), J2 \rightarrow b2\}\}$$

■ V2

$$\{0, -1, 0\}$$

■ L2

$$\frac{y - J2}{-1} == t2$$

■ eP2

$$\{I2, J2 - t2, 0\}$$

$$-(J2 - t2 - b2) == 0$$

$$b2 == J2$$

$$\{I2, b2, 0\}$$

$$\{n2, m2, 0\} == \{I2, J2 - t2, 0\}$$

■ A1

$$A2 = \text{ArcTan}[I3 - I2, J3 - J2]$$

$$T2 = \text{ArcTan}[n2 - I2, m2 - J2]$$

$$B2 = A1 - \pi$$

$$\sin[Q2] == n \sin[R2]$$

$$\sin[T2 - A2] == n \sin[B2 - T2]$$

$$\sin[T2 - A2] == n \sin[A1 - \pi - T2]$$

$$\sin[T2 - A2] == -n \sin[A1 - T2]$$

$$\text{Solve}[\sin[T2 - A2] == -n \sin[A1 - T2], A2]$$

$$A2 \rightarrow T2 + \text{ArcSin}[n \sin[A1 - T2]]$$

■ Point3

$$P3 : \{I3, J3, 0\}$$

$$\text{Solve}[\{J3 == \tan[A2] (I3 - I2) + J2, J3 == b3\}, \{I3, J3\}]$$

$$\{\{I3 \rightarrow -\csc[A2] (-b3 \cos[A2] + J2 \cos[A2] - I2 \sin[A2]), J3 \rightarrow b3\}\}$$