

Kalman Filter

■ 이산 시간 선형 동적 시스템 모델

$$\mathbf{x}_k = \mathbf{F}_k \mathbf{x}_{k-1} + \mathbf{B}_k \mathbf{u}_k + \mathbf{w}_k$$

$\mathbf{x}_k$: 상태 벡터

$\mathbf{F}_k$: 해당 시간에서 이전 상태에 기반한 상태 전이 행렬

$\mathbf{B}_k$: 사용자 입력에 의한 상태 전이 행렬

$\mathbf{u}_k$: 사용자 입력

$\mathbf{w}_k$: 공분산행렬 $\mathbf{Q}_k$ 을 가지는 다변수 정규분포 $\mathbf{w}_k \sim \mathbf{N}(\mathbf{0}, \mathbf{Q}_k)$ 잡음변수

$$\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{v}_k$$

$\mathbf{z}_k$: 상태 벡터 $\mathbf{x}_k$ 와 그 벡터를 측정했을 때 실제로 얻어진 벡터

$\mathbf{H}_k$: 해당 시간에서 측정에 관계되는 행렬

$\mathbf{v}_k$: 공분산행렬 $\mathbf{R}_k$ 을 가진 다변수 정규분포 $\mathbf{v}_k \sim \mathbf{N}(\mathbf{0}, \mathbf{R}_k)$ 잡음변수

초기 상태 $\mathbf{x}_0$ 와 각 잡음 변수 $\mathbf{w}_k, \mathbf{v}_k$ 는 모두 상호 독립이라는 가정이 필요

■ 칼만 필터의 구조

$\hat{\mathbf{x}}_{k|k}$: k시점의 측정 값을 토대로 한 k시점의 상태 추정값

$\mathbf{P}_{k|k}$: k시점의 측정값을 토대로 한 k시점의 상태 공분산행렬

n|m: m시점에서의 측정값을 토대로 한 n시점의 상태 추정을 의미한다.

■ 예측단계 Predict

예측 단계의 계산은 연역적으로 이루어짐

Predicted (a priori) state estimate

연역적 상태 예측 : $\hat{\mathbf{x}}_{k|k-1} = \mathbf{F}_k \hat{\mathbf{x}}_{k-1|k-1} + \mathbf{B}_k \mathbf{u}_k$

Predicted (a priori) estimate covariance

연역적 공분산 예측 : $\mathbf{P}_{k|k-1} = \mathbf{F}_k \mathbf{P}_{k-1|k-1} \mathbf{F}_k^T + \mathbf{Q}_{k-1}$

■ 보정단계 Update

보정단계에서는 앞단계의 예측 값과 실제 측정값간의 오차를 이용해,이전에 얻은 값을 귀납적으로 수정

Innovation or measurement residual

예측 단계와 실제 측정간의 오차 : $\hat{y}_k = z_k - H_k \hat{x}_{k|k-1}$

Innovation (or residual) covariance

예측 단계와 실제 측정간의 오차 공분산 : $S_k = H_k P_{k|k-1} H_k^T + R_k$

Optimal Kalman gain

최적 칼만 게인 (Kalman gain) : $K_k = P_{k|k-1} H_k^T S_k^{-1}$

Updated (a posteriori) state estimate

귀납적 상태 보정 : $\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \hat{y}_k$

Updated (a posteriori) estimate covariance

귀납적 공분산 보정 : $P_{k|k} = (I - K_k H_k) P_{k|k-1}$

■ 불변량 Invariants

모델이 정확하고, $x_{0|0}$ 와 $P_{0|0}$ 값이 정확하게 초기 상태 값의 몫을 반영한다면, 다음 불변량은 보존

$$E[x_k - \hat{x}_{k|k}] = E[x_k - \hat{x}_{k|k-1}] = 0$$

$$E[\hat{y}_k] = 0$$

여기서 $E[\cdot]$ 은 ζ 의 기대값이고, 공분산 매트릭스는 정확하게 추정의 공분산을 반영

$$P_{k|k} = \text{Cov}[x_k - \hat{x}_{k|k}]$$

$$P_{k|k-1} = \text{Cov}[x_k - \hat{x}_{k|k-1}]$$

$$S_k = \text{Cov}[\hat{y}_k]$$

Derivations

■ 연역적 공분산 예측 유도

$$\begin{aligned} P_{k|k-1} &= \text{Cov}[x_k - \hat{x}_{k|k-1}] \\ &= \text{Cov}[(F_k x_{k-1} + B_k u_k + w_k) - (F_k \hat{x}_{k-1|k-1} + B_k u_k)] \\ &= \text{Cov}[F_k (x_{k-1} - \hat{x}_{k-1|k-1}) + w_k] \\ &= F_k \text{Cov}[x_{k-1} - \hat{x}_{k-1|k-1}] F_k^T + \text{Cov}[w_k] \\ &= F_k P_{k-1|k-1} F_k^T + Q_{k-1} \end{aligned}$$

$$\text{Var}[AX + a] = A \text{Var}[X] A^T$$

■ 예측 단계와 실제 측정간의 오차 공분산 유도

$$\begin{aligned} S_k &= \text{Cov}[\hat{y}_k] \\ &= \text{Cov}[z_k - H_k \hat{x}_{k|k-1}] \\ &= \text{Cov}[H_k x_k + v_k - H_k \hat{x}_{k|k-1}] \\ &= \text{Cov}[H_k (x_k - \hat{x}_{k|k-1}) + v_k] \\ &= H_k \text{Cov}[x_k - \hat{x}_{k|k-1}] H_k^T + \text{Cov}[v_k] \\ &= H_k P_{k|k-1} H_k^T + R_k \end{aligned}$$

■ 후부 추정 공분산(posterior estimate covariance) matrix 유도

$$\begin{aligned}
 \mathbf{P}_{k|k} &= \text{Cov}[\mathbf{x}_k - \hat{\mathbf{x}}_{k|k}] \\
 &= \text{Cov}[\mathbf{x}_k - (\hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \mathbf{y}_k)] \\
 &= \text{Cov}[\mathbf{x}_k - (\hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k (\mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_{k|k-1}))] \\
 &= \text{Cov}[\mathbf{x}_k - (\hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k (\mathbf{H}_k \mathbf{x}_k + \mathbf{v}_k - \mathbf{H}_k \hat{\mathbf{x}}_{k|k-1}))] \\
 &= \text{Cov}[(\mathbf{x}_k - \hat{\mathbf{x}}_{k|k-1}) - \mathbf{K}_k \mathbf{H}_k (\mathbf{x}_k - \hat{\mathbf{x}}_{k|k-1}) - \mathbf{K}_k \mathbf{v}_k] \\
 &= \text{Cov}[(\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) (\mathbf{x}_k - \hat{\mathbf{x}}_{k|k-1}) - \mathbf{K}_k \mathbf{v}_k] \\
 &= \text{Cov}[(\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) (\mathbf{x}_k - \hat{\mathbf{x}}_{k|k-1})] + \text{Cov}[\mathbf{K}_k \mathbf{v}_k] \\
 &= (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \text{Cov}[\mathbf{x}_k - \hat{\mathbf{x}}_{k|k-1}] (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k)^T + \mathbf{K}_k \text{Cov}[\mathbf{v}_k] \mathbf{K}_k^T \\
 &= (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_{k|k-1} (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k)^T + \mathbf{K}_k \mathbf{R}_k \mathbf{K}_k^T
 \end{aligned}$$

■ 칼만 이득 $\mathbf{K}_k$ 유도

칼만 필터는 최소 자승 에러 (minimum mean - square error, MMSE) 추정
후부 상태 추정에서 에러

$$\mathbf{x}_k - \hat{\mathbf{x}}_{k|k}$$

에러 자승의 기대값 최소화하는 것을 찾는다

$$\mathbb{E}[(\mathbf{x}_k - \hat{\mathbf{x}}_{k|k})^2]$$

이것은 후부 추정 공분산 매트릭스 $\mathbf{P}_{k|k}$ 의 트레이스 (대각 요소들의 합) 를 최소화하는 것과 같다.

$$\begin{aligned}
 \mathbf{P}_{k|k} &= (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_{k|k-1} (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k)^T + \mathbf{K}_k \mathbf{R}_k \mathbf{K}_k^T \\
 &= \mathbf{P}_{k|k-1} - \mathbf{K}_k \mathbf{H}_k \mathbf{P}_{k|k-1} - \mathbf{P}_{k|k-1} \mathbf{H}_k^T \mathbf{K}_k^T + \mathbf{K}_k \mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^T \mathbf{K}_k^T + \mathbf{K}_k \mathbf{R}_k \mathbf{K}_k^T \\
 &= \mathbf{P}_{k|k-1} - \mathbf{K}_k \mathbf{H}_k \mathbf{P}_{k|k-1} - \mathbf{P}_{k|k-1} \mathbf{H}_k^T \mathbf{K}_k^T + \mathbf{K}_k (\mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^T + \mathbf{R}_k) \mathbf{K}_k^T \\
 &= \mathbf{P}_{k|k-1} - \mathbf{K}_k \mathbf{H}_k \mathbf{P}_{k|k-1} - \mathbf{P}_{k|k-1} \mathbf{H}_k^T \mathbf{K}_k^T + \mathbf{K}_k \mathbf{S}_k \mathbf{K}_k^T
 \end{aligned}$$

그 트레이스는 매트릭스 유도체(매트릭스 미분(calculus))가 0일 때 최소화 됨

$$\begin{aligned}
 \text{tr}(\mathbf{P}_{k|k}) &= \text{tr}(\mathbf{P}_{k|k-1} - \mathbf{K}_k \mathbf{H}_k \mathbf{P}_{k|k-1} - \mathbf{P}_{k|k-1} \mathbf{H}_k^T \mathbf{K}_k^T + \mathbf{K}_k \mathbf{S}_k \mathbf{K}_k^T) \\
 &= \text{tr}(\mathbf{P}_{k|k-1}) - 2 \text{tr}(\mathbf{K}_k \mathbf{H}_k \mathbf{P}_{k|k-1}) + \text{tr}(\mathbf{K}_k \mathbf{S}_k \mathbf{K}_k^T) \\
 \frac{\partial \text{tr}(\mathbf{P}_{k|k})}{\partial \mathbf{K}_k} &= \frac{\partial (\text{tr}(\mathbf{P}_{k|k-1}) - 2 \text{tr}(\mathbf{K}_k \mathbf{H}_k \mathbf{P}_{k|k-1}) + \text{tr}(\mathbf{K}_k \mathbf{S}_k \mathbf{K}_k^T))}{\partial \mathbf{K}_k} \\
 &= \frac{\partial \text{tr}(\mathbf{P}_{k|k-1})}{\partial \mathbf{K}_k} - 2 \frac{\partial \text{tr}(\mathbf{K}_k \mathbf{H}_k \mathbf{P}_{k|k-1})}{\partial \mathbf{K}_k} + \frac{\partial \text{tr}(\mathbf{K}_k \mathbf{S}_k \mathbf{K}_k^T)}{\partial \mathbf{K}_k} \\
 &= \frac{\partial \text{tr}(\mathbf{P}_{k|k-1})}{\partial \mathbf{K}_k} - 2 \frac{\partial \text{tr}(\mathbf{K}_k \mathbf{H}_k \mathbf{P}_{k|k-1})}{\partial \mathbf{K}_k} + \frac{\partial \text{tr}(\mathbf{K}_k \mathbf{S}_k \mathbf{K}_k^T)}{\partial \mathbf{K}_k} \\
 &= -2 \frac{\partial \text{tr}(\mathbf{K}_k \mathbf{H}_k \mathbf{P}_{k|k-1})}{\partial \mathbf{K}_k} + \frac{\partial \text{tr}(\mathbf{K}_k \mathbf{S}_k \mathbf{K}_k^T)}{\partial \mathbf{K}_k} \\
 &= -2 (\mathbf{H}_k \mathbf{P}_{k|k-1})^T + 2 \mathbf{K}_k \mathbf{S}_k
 \end{aligned}$$

$$\frac{\partial \text{tr}(\mathbf{AC})}{\partial \mathbf{A}} = \mathbf{C}^T$$

$$\frac{\partial \text{tr}(\mathbf{ABA}^T)}{\partial \mathbf{A}} = 2 \mathbf{AB}$$

$$\frac{\partial \text{tr}(\mathbf{P}_{k|k})}{\partial \mathbf{K}_k} = -2 (\mathbf{H}_k \mathbf{P}_{k|k-1})^T + 2 \mathbf{K}_k \mathbf{S}_k = \mathbf{0}$$

$$\begin{aligned} \mathbf{K}_k \mathbf{S}_k &= (\mathbf{H}_k \mathbf{P}_{k|k-1})^T = \mathbf{P}_{k|k-1} \mathbf{H}_k^T \\ \mathbf{K}_k &= \mathbf{P}_{k|k-1} \mathbf{H}_k^T \mathbf{S}_k^{-1} \end{aligned}$$

최적의 칼만 이득이라 불리는, 이 이득은 MMSE 추정을 사용했을 때 산출되는 값

■ 후부 에러 공분산(posterior error covariance) 식의 간이화

칼만 이득이 위에서 이끌어 낸 최적의 값과 같을 때 후부 에러 공분산을 단순화할 수 있도록 계산하기 위해 공식을 사용

$\mathbf{S}_k \mathbf{K}_k^T$ 을 칼만 게인 수식의 양쪽 모두에 곱하면

$$\mathbf{K}_k \mathbf{S}_k \mathbf{K}_k^T = \mathbf{P}_{k|k-1} \mathbf{H}_k^T \mathbf{K}_k^T$$

$$\begin{aligned} \mathbf{P}_{k|k} &= \mathbf{P}_{k|k-1} - \mathbf{K}_k \mathbf{H}_k \mathbf{P}_{k|k-1} - \mathbf{P}_{k|k-1} \mathbf{H}_k^T \mathbf{K}_k^T + \mathbf{K}_k \mathbf{S}_k \mathbf{K}_k^T \\ &= \mathbf{P}_{k|k-1} - \mathbf{K}_k \mathbf{H}_k \mathbf{P}_{k|k-1} - \mathbf{P}_{k|k-1} \mathbf{H}_k^T \mathbf{K}_k^T + \mathbf{P}_{k|k-1} \mathbf{H}_k^T \mathbf{K}_k^T \\ &= \mathbf{P}_{k|k-1} - \mathbf{K}_k \mathbf{H}_k \mathbf{P}_{k|k-1} \\ &= (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_{k|k-1} \end{aligned}$$

이 공식은 계산 량이 적고 거의 항상 실제에 사용되지만, 최적의 이득에서 옳을 뿐이다. 만약 수치 정밀도가 수치안정에 문제를 야기하는 것에 대한 영향이 적지만은 않다면, 최적이 아닌 칼만 이득이 고의로 사용된다면, 이 단순화 과정은 지원될 수 없다. 위에서 이끌어낸 후부 에러 공분산 공식을 사용해야만 한다.

EKF - Extended Kalman Filter

$$\mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}) + \mathbf{w}_{k-1}$$

$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}_k) + \mathbf{v}_k$$

$\mathbf{f}$, $\mathbf{h}$ 미분 가능한 함수

■ Predict

$$\text{예측된 상태} : \hat{\mathbf{x}}_{k|k-1} = \mathbf{f}(\hat{\mathbf{x}}_{k-1|k-1}, \mathbf{u}_k)$$

$$\text{예측된 추정 공분산} : \mathbf{P}_{k|k-1} = \mathbf{F}_{k-1} \mathbf{P}_{k-1|k-1} \mathbf{F}_{k-1}^T + \mathbf{Q}_{k-1}$$

■ Update

$$\text{혁신 혹은 추정 나머지} : \mathbf{y}_k = \mathbf{z}_k - \mathbf{h}(\hat{\mathbf{x}}_{k|k-1})$$

$$\text{혁신 혹은 추정 공분산} : \mathbf{S}_k = \mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^T + \mathbf{R}_k$$

$$\text{최적 칼만 게인 (Kalman gain)} : \mathbf{K}_k = \mathbf{P}_{k|k-1} \mathbf{H}_k^T \mathbf{S}_k^{-1}$$

$$\text{업데이트된 상태 추정} : \hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \mathbf{y}_k$$

$$\text{업데이트된 추정 공분산} : \mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_{k|k-1}$$

- 상태 변이와 관측 매트릭스는 자코비안에 따라 정의

$$\mathbf{F}_{k-1} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \bigg|_{\hat{\mathbf{x}}_{k-1|k-1}, \mathbf{u}_k}$$

$$\mathbf{H}_k = \frac{\partial \mathbf{h}}{\partial \mathbf{x}} \bigg|_{\hat{\mathbf{x}}_{k-1|k-1}}$$

EKF_SLAM

EKF_SLAM ($\mathbf{X}_{k-1}$, $\mathbf{P}_{k-1}$, $\mathbf{u}_k$, $\mathbf{z}_k$, $\mathbf{N}_{k-1}$)

$$\mathbf{N}_k = \mathbf{N}_{k-1}$$

$$\mathbf{J}_x = \begin{pmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \end{pmatrix}$$

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{x}_{k-1|k-1} + \mathbf{J}_x^T \begin{pmatrix} -\frac{v_k}{\omega_k} \sin[\mathbf{x}_{k-1, \theta}] + \frac{v_k}{\omega_k} \sin[\mathbf{x}_{k-1, \theta} + w_k \Delta t] \\ \frac{v_k}{\omega_k} \cos[\mathbf{x}_{k-1, \theta}] - \frac{v_k}{\omega_k} \cos[\mathbf{x}_{k-1, \theta} + w_k \Delta t] \\ w_k \Delta t \end{pmatrix}$$

$$\mathbf{F}_k = \mathbf{I} + \mathbf{J}_x^T \begin{pmatrix} 0 & 0 & \frac{v_k}{\omega_k} \cos[\mathbf{x}_{k-1, \theta}] - \frac{v_k}{\omega_k} \cos[\mathbf{x}_{k-1, \theta} + w_k \Delta t] \\ 0 & 0 & \frac{v_k}{\omega_k} \sin[\mathbf{x}_{k-1, \theta}] - \frac{v_k}{\omega_k} \sin[\mathbf{x}_{k-1, \theta} + w_k \Delta t] \\ 0 & 0 & 0 \end{pmatrix} \mathbf{J}_x$$

$$\mathbf{Q}_{k-1} = \mathbf{J}_x^T \mathbf{S} \mathbf{u}_k \mathbf{J}_x$$

$$\mathbf{P}_{k|k-1} = \mathbf{F}_k \mathbf{P}_{k-1|k-1} \mathbf{F}_k^T + \mathbf{J}_x^T \mathbf{S} \mathbf{u}_k \mathbf{J}_x$$

$$\mathbf{R}_k = \begin{pmatrix} \sigma_d & 0 & 0 \\ 0 & \sigma_r & 0 \\ 0 & 0 & \sigma_s \end{pmatrix}$$

for all observed features $\mathbf{z}_k^i = (\mathbf{d}_k^i, \mathbf{r}_k^i, \mathbf{s}_k^i)^T$ do

$$\begin{pmatrix} \hat{\mathbf{x}}_{N_t+1, x} \\ \hat{\mathbf{x}}_{N_t+1, y} \\ \hat{\mathbf{x}}_{N_t+1, s} \end{pmatrix} = \begin{pmatrix} \hat{\mathbf{x}}_{k, x} \\ \hat{\mathbf{x}}_{k, y} \\ \mathbf{s}_k^i \end{pmatrix} + \mathbf{d}_k^i \begin{pmatrix} \cos[\mathbf{r}_k^i + \hat{\mathbf{x}}_{k, \theta}] \\ \sin[\mathbf{r}_k^i + \hat{\mathbf{x}}_{k, \theta}] \\ 0 \end{pmatrix}$$

for $h = 1$ to $N_t + 1$ do

$$\mathbf{P}_{k|k-1} = \mathbf{F}_k \mathbf{P}_{k-1|k-1} \mathbf{F}_k^T + \mathbf{Q}_{k-1}$$

$$\delta_h = \begin{pmatrix} \delta_{h, x} \\ \delta_{h, y} \end{pmatrix} = \begin{pmatrix} \hat{\mathbf{x}}_{h, x} - \hat{\mathbf{x}}_{k|k-1, x} \\ \hat{\mathbf{x}}_{h, y} - \hat{\mathbf{x}}_{k|k-1, y} \end{pmatrix}$$

$$\mathbf{q}_h = \delta_h^T \delta_h$$

$$\mathbf{z}_k^h = \begin{pmatrix} \sqrt{q_h} \\ \text{atan2}(\delta_{h,y}, \delta_{h,x}) - \hat{x}_{k|k-1,\theta} \\ \hat{x}_{k|k-1,s} \end{pmatrix}$$

$$\mathbf{J}_{x,h} = \begin{pmatrix} 1 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 1 & 0 & \dots & 0 \end{pmatrix}$$

$$\mathbf{H}_k^h = \frac{1}{q_h} \begin{pmatrix} \sqrt{q_h} \delta_{h,x} & -\sqrt{q_h} \delta_{h,y} & 0 & -\sqrt{q_h} \delta_{h,x} & \sqrt{q_h} \delta_{h,y} & 0 \\ \delta_{h,y} & \delta_{h,x} & -1 & -\delta_{h,y} & -\delta_{h,x} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \mathbf{J}_{x,h}$$

$$\mathbf{z}_k^h = \mathbf{H}_k^h \hat{\mathbf{x}}_{k|k-1}$$

$$\mathbf{y}_k^h = \mathbf{z}_k^i - \mathbf{H}_k^h \hat{\mathbf{x}}_{k|k-1} = \mathbf{z}_k^i - \mathbf{z}_k^h$$

$$\mathbf{S}_h = \mathbf{H}_k^h \mathbf{P}_{k|k-1} (\mathbf{H}_k^h)^\top + \mathbf{R}_k$$

$$\pi_h = (\mathbf{y}_k^h)^\top (\mathbf{S}_h)^{-1} \mathbf{y}_k^h$$

endfor

$$\pi_{N_k+1} = \alpha$$

$$\mathbf{j}(i) = \underset{h}{\text{argmin}}[\pi_h]$$

$$\mathbf{N}_{k|k} = \text{Max}[\mathbf{N}_{k|k-1}, \mathbf{j}(i)]$$

$$\mathbf{K}_k^i = \mathbf{P}_{k|k-1} (\mathbf{H}_k^{\mathbf{j}(i)})^\top (\mathbf{S}_{\mathbf{j}(i)})^{-1}$$

$$\mathbf{y}_k^{\mathbf{j}(i)} = \mathbf{z}_k^i - \mathbf{z}_k^{\mathbf{j}(i)}$$

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k^i \mathbf{y}_k^{\mathbf{j}(i)}$$

$$\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k^i \mathbf{H}_k^{\mathbf{j}(i)}) \mathbf{P}_{k|k-1}$$

endfor

$$\mathbf{X}_{k|k} = \hat{\mathbf{x}}_{k|k}$$

$$\mathbf{P}_{k|k} = \mathbf{P}_{k|k}$$

Derivations

$$\begin{pmatrix} \mathbf{x}^i \\ \mathbf{y}^i \\ \theta^i \end{pmatrix} = \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \\ \theta \end{pmatrix} + \begin{pmatrix} -\frac{\hat{v}_k}{\hat{\omega}_k} \sin[\theta] + \frac{\hat{v}_k}{\hat{\omega}_k} \sin[\theta + \hat{\omega}_k \Delta t] \\ \frac{\hat{v}_k}{\hat{\omega}_k} \cos[\theta] - \frac{\hat{v}_k}{\hat{\omega}_k} \cos[\theta + \hat{\omega}_k \Delta t] \\ \hat{\omega}_k \Delta t \end{pmatrix}$$

$$\begin{pmatrix} \hat{v}_k \\ \hat{\omega}_k \end{pmatrix} = \begin{pmatrix} \mathbf{v}_k \\ \omega_k \end{pmatrix} + \begin{pmatrix} \varepsilon(\alpha_1 | \mathbf{v}_k | + \alpha_2 | \omega_k |)^2 \\ \varepsilon(\alpha_3 | \mathbf{v}_k | + \alpha_4 | \omega_k |)^2 \end{pmatrix} = \begin{pmatrix} \mathbf{v}_k \\ \omega_k \end{pmatrix} + \mathbf{N}(\mathbf{0}, \mathbf{M}_k)$$

$$\mathbf{x}_k = \begin{pmatrix} \mathbf{x}' \\ \mathbf{y}' \\ \theta' \end{pmatrix} = \mathbf{f}(\mathbf{u}_k, \mathbf{x}_{k-1}) + \mathbf{N}(\mathbf{0}, \mathbf{M}_k) = \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \\ \theta \end{pmatrix} + \begin{pmatrix} -\frac{v_k}{\omega_k} \sin[\theta] + \frac{v_k}{\omega_k} \sin[\theta + \omega_k \Delta t] \\ \frac{v_k}{\omega_k} \cos[\theta] - \frac{v_k}{\omega_k} \cos[\theta + \omega_k \Delta t] \\ \omega_k \Delta t \end{pmatrix} + \mathbf{N}(\mathbf{0}, \mathbf{M}_k)$$

$$\mathbf{f}(\mathbf{u}_k, \mathbf{x}_{k-1}) \approx \mathbf{g}(\mathbf{u}_k, \mathbf{x}_{k-1}) + \mathbf{F}_k (\mathbf{x}_{k-1} - \mathbf{x}_{k-1})$$

$$\mathbf{F}_k = \frac{\partial \mathbf{f}(\mathbf{u}_k, \mathbf{x}_{k-1})}{\partial \mathbf{x}_{k-1}} = \begin{pmatrix} \frac{\partial \mathbf{x}'}{\partial \mathbf{x}_{k-1, x}} & \frac{\partial \mathbf{x}'}{\partial \mathbf{x}_{k-1, y}} & \frac{\partial \mathbf{x}'}{\partial \mathbf{x}_{k-1, \theta}} \\ \frac{\partial \mathbf{y}'}{\partial \mathbf{x}_{k-1, x}} & \frac{\partial \mathbf{y}'}{\partial \mathbf{x}_{k-1, y}} & \frac{\partial \mathbf{y}'}{\partial \mathbf{x}_{k-1, \theta}} \\ \frac{\partial \theta'}{\partial \mathbf{x}_{k-1, x}} & \frac{\partial \theta'}{\partial \mathbf{x}_{k-1, y}} & \frac{\partial \theta'}{\partial \mathbf{x}_{k-1, \theta}} \end{pmatrix}$$

$$\mathbf{F}_k = \begin{pmatrix} 1 & 0 & \frac{v_k}{\omega_k} (-\cos[\mathbf{x}_{k-1, \theta}] + \cos[\mathbf{x}_{k-1, \theta} + \omega_k \Delta t]) \\ 0 & 1 & \frac{v_k}{\omega_k} (-\sin[\mathbf{x}_{k-1, \theta}] + \sin[\mathbf{x}_{k-1, \theta} + \omega_k \Delta t]) \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{M}_k = \begin{pmatrix} (\alpha_1 | v_k | + \alpha_2 | \omega_k |)^2 & 0 \\ 0 & (\alpha_3 | v_k | + \alpha_4 | \omega_k |)^2 \end{pmatrix}$$

$$\mathbf{v}_k = \frac{\partial \mathbf{f}(\mathbf{u}_k, \mathbf{x}_{k-1})}{\partial \mathbf{u}_k} = \begin{pmatrix} \frac{\partial \mathbf{x}'}{\partial v_k} & \frac{\partial \mathbf{x}'}{\partial \omega_k} \\ \frac{\partial \mathbf{y}'}{\partial v_k} & \frac{\partial \mathbf{y}'}{\partial \omega_k} \\ \frac{\partial \theta'}{\partial v_k} & \frac{\partial \theta'}{\partial \omega_k} \end{pmatrix} = \begin{pmatrix} -\frac{\sin[\theta] + \sin[\theta + \omega_k \Delta t]}{\omega_k} & \frac{v_k (\sin[\theta] - \sin[\theta + \omega_k \Delta t])}{\omega_k^2} + \frac{v_k \cos[\theta + \omega_k \Delta t] \Delta t}{\omega_k^2} \\ \frac{\cos[\theta] - \cos[\theta + \omega_k \Delta t]}{\omega_k} & -\frac{v_k (\cos[\theta] - \cos[\theta + \omega_k \Delta t])}{\omega_k^2} + \frac{v_k \sin[\theta + \omega_k \Delta t] \Delta t}{\omega_k^2} \\ 0 & \Delta t \end{pmatrix}$$

$$\mathbf{z}_k^i = \begin{pmatrix} \mathbf{d}_k^i \\ \mathbf{r}_k^i \\ \mathbf{s}_k^i \end{pmatrix} = \mathbf{h}(\mathbf{x}_k, \mathbf{j}, \mathbf{m}) = \begin{pmatrix} \sqrt{(\mathbf{m}_{j, x} - \mathbf{x})^2 + (\mathbf{m}_{j, y} - \mathbf{y})^2} \\ \text{atan2}(\mathbf{m}_{j, y} - \mathbf{y}, \mathbf{m}_{j, x} - \mathbf{x}) - \theta \\ \mathbf{m}_{j, s} \end{pmatrix} + \mathbf{N}(\mathbf{0}, \mathbf{Q}_k)$$

$$\mathbf{H}_k^i = \frac{\partial \mathbf{h}(\hat{\mathbf{x}}_k, \mathbf{j}, \mathbf{m})}{\partial \mathbf{x}_k} = \begin{pmatrix} \frac{\partial \mathbf{d}_k^i}{\partial \mathbf{x}_{k, x}} & \frac{\partial \mathbf{d}_k^i}{\partial \mathbf{x}_{k, y}} & \frac{\partial \mathbf{d}_k^i}{\partial \mathbf{x}_{k, \theta}} \\ \frac{\partial \mathbf{r}_k^i}{\partial \mathbf{x}_{k, x}} & \frac{\partial \mathbf{r}_k^i}{\partial \mathbf{x}_{k, y}} & \frac{\partial \mathbf{r}_k^i}{\partial \mathbf{x}_{k, \theta}} \\ \frac{\partial \mathbf{s}_k^i}{\partial \mathbf{x}_{k, x}} & \frac{\partial \mathbf{s}_k^i}{\partial \mathbf{x}_{k, y}} & \frac{\partial \mathbf{s}_k^i}{\partial \mathbf{x}_{k, \theta}} \end{pmatrix} = \begin{pmatrix} -\frac{\mathbf{m}_{j, x} - \mathbf{x}_{k, x}}{\sqrt{q}} & -\frac{\mathbf{m}_{j, y} - \mathbf{x}_{k, y}}{\sqrt{q}} & 0 \\ \frac{\mathbf{m}_{j, y} - \mathbf{x}_{k, y}}{q} & \frac{\mathbf{m}_{j, x} - \mathbf{x}_{k, x}}{q} & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{Q}_k = \begin{pmatrix} \sigma_d^2 & 0 & 0 \\ 0 & \sigma_r^2 & 0 \\ 0 & 0 & \sigma_s^2 \end{pmatrix}$$

EKF myslam

Kinematic_Model (실제로봇 위치 이동)

Predict_Process (가상로봇 위치 이동)

1 초에 한번씩

Observe (실제로봇과 비콘들 측정)

(실제로봇 기준으로 거리가 20 이하인 비콘만 로봇을 기준으로 켜진 거리, 각도, ID return)

측정된 비콘 중 한개씩

측정된 비콘 중 저장된 Landmark 면

Update_Association (해당 Landmark만 연산)

측정된 closest Landmark들과 저장된 Landmark 와 비교

Find_Nearest_Landmark

(측정된 비콘이 가상 위치에서 Landmark와

거리가 가까운 Landmark 검색 closest Landmark들 return)

Chi - Square Test (측정된 closest Landmark 들을 queue 저장)

Queue 에 저장된 값중 작은 값을 Landmark 로 가정

Update_Association (가정된 Landmark만 연산)

가능성이 없으면 Landmark 추가

Add_Landmark (측정된 비콘 거리, 각도, ID)

Update_Association (측정된 비콘만 연산)

실제로봇, 가상로봇 위치 표기

실제로봇과 관측된 비콘만 연결선 표시

Covariance (가상로봇, 가상 비콘) 표시

Landmark 표시

Algorithm

dt = 1 / Hz

$$\frac{dF}{du} = Ju = dt * \begin{pmatrix} \cos[r_\theta] & 0 \\ \sin[r_\theta] & 0 \\ 0 & 1 \end{pmatrix}$$

$$\frac{dF}{dx} = F_k = \begin{pmatrix} 1 & 0 & -dt * u_1 \sin[\theta] \\ 0 & 1 & dt * u_1 \cos[\theta] \\ 0 & 0 & 1 \end{pmatrix}$$

■ Kinematic_Model (실제로봇 위치 이동)

■ $X_k = F_k X_{k-1} + B_k u_k + w_k$

$$F_k = \begin{pmatrix} 1 & 0 & -dt * u_{dist} * \sin[k_\theta] \\ 0 & 1 & dt * u_{dist} * \cos[k_\theta] \\ 0 & 0 & 1 \end{pmatrix}$$

$$X_r = \begin{pmatrix} X_{r_x} \\ X_{r_y} \\ X_{r_\theta} \end{pmatrix}$$

$$\mathbf{B}_k = dt * \begin{pmatrix} \cos[Xr_\theta] + Xr_\theta \sin[Xr_\theta] & 0 \\ \sin[Xr_\theta] - Xr_\theta \cos[Xr_\theta] & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mathbf{u}_k = \begin{pmatrix} u_{\text{dist}} \\ u_{\text{rad}} \end{pmatrix}$$

$$\text{Random} = (Rd_1 \quad Rd_2 \quad Rd_3)$$

$$\mathbf{J}_u = dt * \begin{pmatrix} \cos[Xr_\theta] & 0 \\ \sin[Xr_\theta] & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mathbf{w} = \begin{pmatrix} w_{\text{dist}} \\ w_{\text{rad}} \end{pmatrix}$$

$$\mathbf{w}_k = \text{Random} * \mathbf{J}_u * \mathbf{w}$$

$$\mathbf{X}_r = \mathbf{F}_k \mathbf{X}_{r-1} + \mathbf{B}_k \mathbf{u}_k + \mathbf{w}_k$$

$$= \begin{pmatrix} 1 & 0 & -dt * u_{\text{dist}} * \sin[Xr_\theta] \\ 0 & 1 & dt * u_{\text{dist}} * \cos[Xr_\theta] \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} X_{r_x} \\ X_{r_y} \\ X_{r_\theta} \end{pmatrix} +$$

$$dt * \begin{pmatrix} \cos[Xr_\theta] + Xr_\theta \sin[Xr_\theta] & 0 \\ \sin[Xr_\theta] - Xr_\theta \cos[Xr_\theta] & 0 \\ 0 & 1 \end{pmatrix} * \begin{pmatrix} u_{\text{dist}} \\ u_{\text{rad}} \end{pmatrix} + dt * (Rd_1 \quad Rd_2 \quad Rd_3) * \begin{pmatrix} \cos[Xr_\theta] & 0 \\ \sin[Xr_\theta] & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} w_{\text{dist}} \\ w_{\text{rad}} \end{pmatrix}$$

$$\mathbf{X}_r = \mathbf{X}_r + dt * \begin{pmatrix} u_{\text{dist}} \cos[r_\theta] \\ u_{\text{dist}} \sin[r_\theta] \\ u_{\text{rad}} \end{pmatrix} + dt * \begin{pmatrix} Rd_1 w_{\text{dist}} \cos[r_\theta] \\ Rd_2 w_{\text{dist}} \sin[r_\theta] \\ Rd_3 w_{\text{rad}} \end{pmatrix}$$

■ Predict_Process (가상로봇 위치 이동)

$$\hat{\mathbf{X}}_{k|k-1} = \mathbf{F}_k \hat{\mathbf{X}}_{k-1|k-1} + \mathbf{B}_k \mathbf{u}_k$$

$$\mathbf{P}_{k|k-1} = \mathbf{F}_k \mathbf{P}_{k-1|k-1} \mathbf{F}_k^T + \mathbf{Q}_{k-1}$$

$$\blacksquare \hat{\mathbf{X}}_{k|k-1} = \mathbf{F}_k \hat{\mathbf{X}}_{k-1|k-1} + \mathbf{B}_k \mathbf{u}_k$$

$$\hat{\mathbf{X}}_k = \begin{pmatrix} \begin{pmatrix} Xk_x \\ Xk_y \\ Xk_\theta \end{pmatrix} \\ \begin{pmatrix} L1_x \\ L1_y \end{pmatrix} \\ \begin{pmatrix} L2_x \\ L2_y \end{pmatrix} \\ \dots \\ \begin{pmatrix} Ln_x \\ Ln_y \end{pmatrix} \end{pmatrix}$$

$$\mathbf{F}_k = \begin{pmatrix} 1 & 0 & -dt * u_{\text{dist}} * \sin[k_\theta] \\ 0 & 1 & dt * u_{\text{dist}} * \cos[k_\theta] \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{B}_k = dt * \begin{pmatrix} \cos[Xr_\theta] + Xr_\theta \sin[Xr_\theta] & 0 \\ \sin[Xr_\theta] - Xr_\theta \cos[Xr_\theta] & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mathbf{u}_k = \begin{pmatrix} \mathbf{u}_{\text{dist}} \\ \mathbf{u}_{\text{rad}} \end{pmatrix}$$

$$\hat{\mathbf{x}}_{k|k-1} = \begin{pmatrix} 1 & 0 & -dt * \mathbf{u}_{\text{dist}} * \sin[k_\theta] \\ 0 & 1 & dt * \mathbf{u}_{\text{dist}} * \cos[k_\theta] \\ 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} \mathbf{x}k_x \\ \mathbf{x}k_y \\ \mathbf{x}k_\theta \end{pmatrix} + dt * \begin{pmatrix} \cos[Xr_\theta] + Xr_\theta \sin[Xr_\theta] & 0 \\ \sin[Xr_\theta] - Xr_\theta \cos[Xr_\theta] & 0 \\ 0 & 1 \end{pmatrix} * \begin{pmatrix} \mathbf{u}_{\text{dist}} \\ \mathbf{u}_{\text{rad}} \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{x}k_x \\ \mathbf{x}k_y \\ \mathbf{x}k_\theta \end{pmatrix} = \begin{pmatrix} \mathbf{x}k_x \\ \mathbf{x}k_y \\ \mathbf{x}k_\theta \end{pmatrix} + dt * \begin{pmatrix} \mathbf{u}_{\text{dist}} \cos[k_\theta] \\ \mathbf{u}_{\text{dist}} \sin[k_\theta] \\ \mathbf{u}_{\text{rad}} \end{pmatrix}$$

$$\mathbf{P}_{k|k-1} = \mathbf{F}_k \mathbf{P}_{k-1|k-1} \mathbf{F}_k^T + \mathbf{Q}_{k-1}$$

$$\mathbf{P}_k = \begin{pmatrix} \begin{pmatrix} \mathbf{P}_{1,1} & \mathbf{P}_{1,2} & \mathbf{P}_{1,3} \\ \mathbf{P}_{2,1} & \mathbf{P}_{2,2} & \mathbf{P}_{2,3} \\ \mathbf{P}_{3,1} & \mathbf{P}_{3,2} & \mathbf{P}_{3,3} \end{pmatrix} & \dots & \dots & \dots \\ \vdots & \ddots & \vdots & \dots \\ \vdots & \dots & \begin{pmatrix} \mathbf{p}_{3+2(N-1)+1, 3+2(N-1)+1} & \mathbf{p}_{3+2(N-1)+1, 3+2(N-1)+2} \\ \mathbf{p}_{3+2(N-1)+2, 3+2(N-1)+1} & \mathbf{p}_{3+2(N-1)+2, 3+2(N-1)+2} \end{pmatrix} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$\mathbf{P}_k = \begin{pmatrix} \begin{pmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 15 * \frac{\pi}{180} \end{pmatrix} & \begin{pmatrix} 10^6 & 0 \\ 0 & 10^6 \end{pmatrix} & \dots & \dots & \dots & \dots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \end{pmatrix}$$

$$\mathbf{F}_k = \begin{pmatrix} 1 & 0 & -dt * \mathbf{u}_{\text{dist}} * \sin[k_\theta] \\ 0 & 1 & dt * \mathbf{u}_{\text{dist}} * \cos[k_\theta] \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{P}_{1 \times 3 \times n} = \begin{pmatrix} \mathbf{P}_x & \square & \square & \square & \square & \square & \square & \square & \square & \square \\ \square & \mathbf{P}_y & \square & \square & \square & \square & \square & \square & \square & \square \\ \square & \square & \mathbf{P}_\theta & \square & \square & \square & \square & \square & \square & \square \end{pmatrix}$$

$$\mathbf{P}_{2 \times n-3 \times n} = \begin{pmatrix} \square & \square & \square & \mathbf{P}_{L1x} & \square & \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \mathbf{P}_{L1y} & \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square & \mathbf{P}_{L2x} & \square & \square & \square & \square \\ \square & \square & \square & \square & \square & \square & \mathbf{P}_{L2y} & \square & \square & \square \\ \square & \square & \square & \square & \square & \square & \square & \ddots & \square & \square \\ \square & \square & \square & \square & \square & \square & \square & \square & \mathbf{P}_{Ln_x} & \square \\ \square & \square & \square & \square & \square & \square & \square & \square & \square & \mathbf{P}_{Ln_y} \end{pmatrix}$$

$$\mathbf{Aux}_{n \times n} = \begin{pmatrix} \mathbf{F}_k * \mathbf{P}_1 \\ \mathbf{P}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{Aux1}_{3 \times 3} & \mathbf{Aux2}_{3 \times n-3} \\ \mathbf{Aux3}_{n-3 \times 3} & \mathbf{Aux4}_{n-3 \times n-3} \end{pmatrix}$$

$$\mathbf{T}_A = \begin{pmatrix} \mathbf{Aux1} * \mathbf{F}_k^T & \mathbf{Aux2} \\ \mathbf{Aux3} * \mathbf{F}_k^T & \mathbf{Aux4} \end{pmatrix}$$

$$\mathbf{Q}_{k-1} = \mathbf{J} \mathbf{u} * \mathbf{S} \mathbf{u} * \mathbf{J}^T$$

$$\begin{aligned}
\mathbf{J}_u &= dt * \begin{pmatrix} \cos[k_\theta] & 0 \\ \sin[k_\theta] & 0 \\ 0 & 1 \end{pmatrix} \\
\mathbf{S}_u &= \mathbf{I} w^2 = \begin{pmatrix} w_{\text{dist}}^2 & 0 \\ 0 & w_{\text{rad}}^2 \end{pmatrix} \\
\mathbf{T}_B &= \begin{pmatrix} \mathbf{J}_u * \mathbf{S}_u * \mathbf{J}_u^T & \mathbf{Zeros}_{3 \times n-3} \\ \mathbf{Zeros}_{n-3 \times 3} & \mathbf{Zeros}_{n-3 \times n-3} \end{pmatrix} \\
\mathbf{P}_k &= \mathbf{T}_A + \mathbf{T}_B \\
\mathbf{P}_k &= \mathbf{F}_k * \mathbf{P}_k * \mathbf{F}_k^T + \mathbf{J}_u * \mathbf{S}_u * \mathbf{J}_u^T
\end{aligned}$$

■ Observe (실제로봇과 비콘들 측정)

$$\begin{aligned}
\mathbf{S}_m &= \begin{pmatrix} \mathbf{S}_{m_{\text{dist}}} \\ \mathbf{S}_{m_{\text{rad}}} \end{pmatrix} \\
\mathbf{z}_k &= \begin{pmatrix} \mathbf{z}_{\text{dist}} \\ \mathbf{z}_{\text{rad}} \end{pmatrix} = \begin{pmatrix} \sqrt{(b_x - x_{r_x})^2 + (b_y - x_{r_y})^2} \\ \text{ArcTan}[b_x - x_{r_x}, b_y - x_{r_y}] - r_\theta \end{pmatrix} + \text{Random} * \begin{pmatrix} \mathbf{S}_{m_{\text{dist}}} \\ \mathbf{S}_{m_{\text{rad}}} \end{pmatrix} \\
\mathbf{z}_k &= \begin{pmatrix} \mathbf{z}_{\text{dist}} \\ \mathbf{z}_{\text{rad}} \end{pmatrix} = \begin{pmatrix} \sqrt{(b_x - x_{r_x})^2 + (b_y - x_{r_y})^2} + \mathbf{Rd}_1 * \mathbf{S}_{m_{\text{dist}}} \\ \text{ArcTan}[b_x - x_{r_x}, b_y - x_{r_y}] - r_\theta + \mathbf{Rd}_2 * \mathbf{S}_{m_{\text{rad}}} \end{pmatrix} \\
\mathbf{z}_n &= \mathbf{b}_{\text{id}}
\end{aligned}$$

■ Find_Nearest_Landmark

$$\begin{aligned}
\text{dist} &= \sqrt{(x_{k_x} + z_{\text{dist}} * \cos[z_{\text{rad}} + x_{k_\theta}] - l_{n_x})^2 + (x_{k_y} + z_{\text{dist}} * \sin[z_{\text{rad}} + x_{k_\theta}] - l_{n_y})^2} \\
\text{closest} &= \text{dist} < \text{min_dist}
\end{aligned}$$

■ chi-square test

$$\begin{aligned}
\mathbf{q} &= \hat{\mathbf{y}}_k^T * \mathbf{S}^{-1} * \hat{\mathbf{y}}_k \\
\mathbf{q1} &= \mathbf{q} + \text{Log}[\text{Det}[\mathbf{S}]] \\
\text{Min}[\mathbf{q1}]
\end{aligned}$$

■ Add_Landmark (측정된 비콘 거리,각도,비콘명)

$$\mathbf{X}_k = \begin{pmatrix} \begin{pmatrix} x_{k_x} \\ x_{k_y} \\ x_{k_\theta} \end{pmatrix} \\ \begin{pmatrix} l_{1_x} \\ l_{1_y} \end{pmatrix} \\ \begin{pmatrix} l_{2_x} \\ l_{2_y} \end{pmatrix} \\ \begin{pmatrix} x_{k_x} + z_{\text{dist}} * \cos[z_{\text{rad}} + x_{k_\theta}] \\ x_{k_y} + z_{\text{dist}} * \sin[z_{\text{rad}} + x_{k_\theta}] \\ \vdots \end{pmatrix} \end{pmatrix}$$

$$P_k = \begin{pmatrix} \begin{pmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 15 * \frac{\pi}{180} \end{pmatrix} & \begin{pmatrix} \square & \square \\ \square & \square \\ \square & \square \end{pmatrix} & \begin{pmatrix} \square & \square \\ \square & \square \\ \square & \square \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} \square & \square \\ \square & \square \\ \square & \square \end{pmatrix} \\ \begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix} & \begin{pmatrix} 10^6 & \square \\ \square & 10^6 \end{pmatrix} & \begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix} \\ \begin{pmatrix} \square & \square & \square \\ \square & \square & \square \end{pmatrix} & \begin{pmatrix} 10^6 & \square \\ \square & 10^6 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 10^6 & 0 \\ 0 & 10^6 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} \square & \square & \square \\ \square & \square & \square \end{pmatrix} & \begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix} & \begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \ddots \\ \begin{pmatrix} \square & \square & \square \\ \square & \square & \square \end{pmatrix} & \begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix} & \begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \end{pmatrix}$$

■ Update_Association (해당 비콘만 연상)

$$\hat{y}_k = z_k - H_k \hat{x}_{k|k-1}$$

$$S_k = H_k P_{k|k-1} H_k^T + R_k$$

$$K_k = P_{k|k-1} H_k^T S_k^{-1}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \hat{y}_k$$

$$P_{k|k} = (I - K_k H_k) P_{k|k-1}$$

■ $\hat{y}_k = z_k - H_k \hat{x}_{k|k-1}$

$$z_k = \begin{pmatrix} z_{\text{dist}} \\ z_{\text{rad}} \end{pmatrix}$$

$$D = \begin{pmatrix} dx \\ dy \end{pmatrix} = \begin{pmatrix} Xk_x - Ln_x \\ Xk_y - Ln_y \end{pmatrix}$$

$$d = D^T D = dx^2 + dy^2$$

$$H_k = \begin{pmatrix} \frac{dx}{\sqrt{d}} & \frac{dy}{\sqrt{d}} & 0 & 0 & 0 & 0 & \frac{-dx}{\sqrt{d}} & \frac{-dy}{\sqrt{d}} & 0 & 0 \\ \frac{-dy}{d} & \frac{dx}{d} & 0 & 0 & 0 & 0 & \frac{dy}{d} & \frac{-dx}{d} & 0 & 0 \end{pmatrix}$$

$$\hat{x}_k = \begin{pmatrix} \begin{pmatrix} Xk_x \\ Xk_y \\ Xk_\theta \end{pmatrix} \\ \begin{pmatrix} L1_x \\ L1_y \end{pmatrix} \\ \begin{pmatrix} L2_x \\ L2_y \end{pmatrix} \\ \dots \\ \begin{pmatrix} Ln_x \\ Ln_y \end{pmatrix} \end{pmatrix}$$

$$\mathbf{H}_k \hat{\mathbf{x}}_k = \begin{pmatrix} \frac{dx}{\sqrt{d}} & \frac{dy}{\sqrt{d}} & 0 & 0 & 0 & 0 & \frac{-dx}{\sqrt{d}} & \frac{-dy}{\sqrt{d}} & 0 & 0 \\ \frac{-dy}{d} & \frac{dx}{d} & 0 & 0 & 0 & 0 & \frac{dy}{d} & \frac{-dx}{d} & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{xk}_x \\ \mathbf{xk}_y \\ \mathbf{xk}_\theta \\ \vdots \\ \vdots \\ \vdots \\ \mathbf{Ln}_x \\ \mathbf{Ln}_y \\ \vdots \\ \vdots \end{pmatrix}$$

$$= \begin{pmatrix} \frac{dx}{\sqrt{d}} (\mathbf{xk}_x - \mathbf{Ln}_x) + \frac{dy}{\sqrt{d}} (\mathbf{xk}_y - \mathbf{Ln}_y) \\ \frac{dy}{d} (-\mathbf{xk}_x + \mathbf{Ln}_x) + \frac{dx}{d} (-\mathbf{xk}_y + \mathbf{Ln}_y) \end{pmatrix}$$

$$= \begin{pmatrix} \frac{(\mathbf{xk}_x - \mathbf{Ln}_x)^2 + (\mathbf{xk}_y - \mathbf{Ln}_y)^2}{\sqrt{dx^2 + dy^2}} \\ \frac{(\mathbf{xk}_y - \mathbf{Ln}_y) (-\mathbf{xk}_x + \mathbf{Ln}_x) + (\mathbf{xk}_x - \mathbf{Ln}_x) (-\mathbf{xk}_y + \mathbf{Ln}_y)}{d} \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt{dx^2 + dy^2} \\ -2 \frac{(\mathbf{xk}_x - \mathbf{Ln}_x) (\mathbf{xk}_y - \mathbf{Ln}_y)}{dx^2 + dy^2} \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt{d} \\ -2 \frac{dx dy}{d} \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{H}_{\text{dist}} \\ \mathbf{H}_{\text{rad}} \end{pmatrix} = \begin{pmatrix} \sqrt{d} \\ \text{ArcTan}[dy, dx] - \mathbf{xk}_\theta \end{pmatrix}$$

$$\hat{\mathbf{y}}_k = \mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_{k|k-1} = \begin{pmatrix} \mathbf{z}_{\text{dist}} \\ \mathbf{z}_{\text{rad}} \end{pmatrix} - \begin{pmatrix} \sqrt{d} \\ -2 \frac{dx dy}{d} \end{pmatrix}$$

$$\hat{\mathbf{y}}_k = \begin{pmatrix} \mathbf{z}_{\text{dist}} - \mathbf{H}_{\text{dist}} \\ \mathbf{z}_{\text{rad}} - \mathbf{H}_{\text{rad}} \end{pmatrix}$$

$$\blacksquare \mathbf{S}_k = \mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^T + \mathbf{R}_k$$

$$\mathbf{R}_k = \mathbf{I} \mathbf{S} m^2 = \begin{pmatrix} \mathbf{S} m_{\text{dist}}^2 & 0 \\ 0 & \mathbf{S} m_{\text{rad}}^2 \end{pmatrix}$$

$$\blacksquare K_k = P_{k|k-1} H_k^T S_k^{-1}$$

$$K_k = P_k * H_k^T * S^{-1}$$

$$= \begin{pmatrix} \begin{pmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 15 * \frac{\pi}{180} \end{pmatrix} & \begin{pmatrix} \square & \square \\ \square & \square \\ \square & \square \end{pmatrix} & \begin{pmatrix} \square & \square \\ \square & \square \\ \square & \square \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} \square & \square \\ \square & \square \\ \square & \square \end{pmatrix} \\ \begin{pmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{pmatrix} & \begin{pmatrix} 10^6 & \square \\ \square & 10^6 \end{pmatrix} & \begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix} \\ \begin{pmatrix} \square & \square & \square \\ \square & \square & \square \end{pmatrix} & \begin{pmatrix} 10^6 & \square \\ \square & 10^6 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \begin{pmatrix} 10^6 & 0 \\ 0 & 10^6 \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} \square & \square & \square \\ \square & \square & \square \end{pmatrix} & \begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix} & \begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix} & \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} & \ddots \end{pmatrix}$$

$$\begin{pmatrix} \frac{dx}{\sqrt{d}} & \frac{-dy}{d} \\ \frac{dy}{\sqrt{d}} & \frac{dx}{d} \\ 0 & -1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{-dx}{\sqrt{d}} & \frac{dy}{d} \\ \frac{-dy}{\sqrt{d}} & \frac{-dx}{d} \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{S_{2,2}}{-S_{1,2} S_{2,1} + S_{1,1} S_{2,2}} & -\frac{S_{1,2}}{-S_{1,2} S_{2,1} + S_{1,1} S_{2,2}} \\ -\frac{S_{2,1}}{-S_{1,2} S_{2,1} + S_{1,1} S_{2,2}} & \frac{S_{1,1}}{-S_{1,2} S_{2,1} + S_{1,1} S_{2,2}} \end{pmatrix}$$

$$= \begin{pmatrix} K_{1,1} & K_{1,2} \\ K_{2,1} & K_{2,2} \\ \vdots & \vdots \\ \vdots & \vdots \\ \vdots & \vdots \\ \vdots & \vdots \\ \vdots & \vdots \\ \vdots & \vdots \\ \vdots & \vdots \\ \vdots & \vdots \end{pmatrix}$$

$$\blacksquare P_{k|k} = (I - K_k H_k) P_{k|k-1}$$

$$\begin{aligned} P_k &= (I - K_k H_k) P_k = P_k - K_k H_k P_k = P_k - K_k * S * K_k^T \\ &= (n \times n) - (n \times 2) * (2 \times 2) * (2 \times n) \end{aligned}$$

$$\hat{\mathbf{X}}_{k|k} = \hat{\mathbf{X}}_{k|k-1} + \mathbf{K}_k \hat{\mathbf{Y}}_k$$

$$\mathbf{X}_k = \mathbf{X}_k + \mathbf{K}_k * \hat{\mathbf{Y}}_k$$

$$= \begin{pmatrix} \begin{pmatrix} k_x \\ k_y \\ k_\theta \end{pmatrix} \\ \begin{pmatrix} L1_x \\ L1_y \end{pmatrix} \\ \begin{pmatrix} L2_x \\ L2_y \end{pmatrix} \\ \begin{pmatrix} k_x + z_d * \cos[z_r + k_\theta] \\ k_x + z_d * \sin[z_r + k_\theta] \\ \vdots \end{pmatrix} \end{pmatrix} + \begin{pmatrix} K_{1,1} & K_{1,2} \\ K_{2,1} & K_{2,2} \\ \vdots & \vdots \\ \vdots & \vdots \\ \vdots & \vdots \\ \vdots & \vdots \\ \vdots & \vdots \\ \vdots & \vdots \\ \vdots & \vdots \end{pmatrix} \begin{pmatrix} z_d - H_d \\ z_r - H_r \end{pmatrix}$$

Covariance

■ 공분산 - 상관관계

$$\text{Cov}[\mathbf{X}, \mathbf{Y}] = \mathbf{E}[(\mathbf{X} - \mathbf{E}[\mathbf{X}]) (\mathbf{Y} - \mathbf{E}[\mathbf{Y}])]$$

$$\text{Cov}[\mathbf{X}, \mathbf{Y}] = \mathbf{E}[\mathbf{X} \mathbf{Y}] - \mathbf{E}[\mathbf{X}] \mathbf{E}[\mathbf{Y}]$$

$$\text{Cov}[\mathbf{X}, \mathbf{Y}] = \mathbf{E}[(\mathbf{X} - \mathbf{E}[\mathbf{X}]) (\mathbf{Y} - \mathbf{E}[\mathbf{Y}])^T]$$

$$\text{Cov}[\mathbf{X}, \mathbf{Y}] = \mathbf{E}[\mathbf{X} \mathbf{Y}^T] - \mathbf{E}[\mathbf{X}] \mathbf{E}[\mathbf{Y}]^T$$

$$\text{Cov}[\mathbf{X}, \mathbf{0}] = \mathbf{0}$$

$$\text{Cov}[\mathbf{X}, \mathbf{X}] = \text{Var}[\mathbf{X}]$$

$$\text{Cov}[\mathbf{X}, \mathbf{Y}] = \text{Cov}[\mathbf{Y}, \mathbf{X}]$$

$$\text{Cov}[\mathbf{aX}, \mathbf{bY}] = \mathbf{ab} \text{Cov}[\mathbf{X}, \mathbf{Y}]$$

$$\text{Cov}[\mathbf{X} + \mathbf{a}, \mathbf{Y} + \mathbf{b}] = \text{Cov}[\mathbf{X}, \mathbf{Y}]$$

$$\text{Cov}[\mathbf{aX} + \mathbf{bY}, \mathbf{cW} + \mathbf{dV}] = \mathbf{ac} \text{Cov}[\mathbf{X}, \mathbf{W}] + \mathbf{ad} \text{Cov}[\mathbf{X}, \mathbf{V}] + \mathbf{bc} \text{Cov}[\mathbf{Y}, \mathbf{W}] + \mathbf{bd} \text{Cov}[\mathbf{Y}, \mathbf{V}]$$

$$\text{Cov}[\mathbf{X}_i, \mathbf{X}_j] = \mathbf{E}[(\mathbf{X}_i - \mathbf{E}[\mathbf{X}_i]) (\mathbf{X}_j - \mathbf{E}[\mathbf{X}_j])] = \mathbf{E}[\mathbf{X}_i \mathbf{X}_j] - \mathbf{E}[\mathbf{X}_i] \cdot \mathbf{E}[\mathbf{X}_j]$$

■ 공분산행렬

$$\mathbf{X} = \begin{pmatrix} X_1 \\ \vdots \\ X_n \end{pmatrix}$$

$$\sum_{i,j} = \text{Cov}[\mathbf{X}_i, \mathbf{X}_j] = \mathbf{E}[(\mathbf{X}_i - \mathbf{E}[\mathbf{X}_i]) (\mathbf{X}_j - \mathbf{E}[\mathbf{X}_j])] = \mathbf{E}[(\mathbf{X}_i - \mu_i) (\mathbf{X}_j - \mu_j)]$$

$$\sum = \begin{pmatrix} \mathbf{E}[(\mathbf{X}_1 - \mu_1) (\mathbf{X}_1 - \mu_1)] & \mathbf{E}[(\mathbf{X}_1 - \mu_1) (\mathbf{X}_2 - \mu_2)] & \cdots & \mathbf{E}[(\mathbf{X}_1 - \mu_1) (\mathbf{X}_n - \mu_n)] \\ \mathbf{E}[(\mathbf{X}_2 - \mu_2) (\mathbf{X}_1 - \mu_1)] & \mathbf{E}[(\mathbf{X}_2 - \mu_2) (\mathbf{X}_2 - \mu_2)] & \cdots & \mathbf{E}[(\mathbf{X}_2 - \mu_2) (\mathbf{X}_n - \mu_n)] \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{E}[(\mathbf{X}_n - \mu_n) (\mathbf{X}_1 - \mu_1)] & \mathbf{E}[(\mathbf{X}_n - \mu_n) (\mathbf{X}_2 - \mu_2)] & \cdots & \mathbf{E}[(\mathbf{X}_n - \mu_n) (\mathbf{X}_n - \mu_n)] \end{pmatrix}$$

$$\sum = \mathbf{E}[(\mathbf{X} - \mathbf{E}[\mathbf{X}]) (\mathbf{X} - \mathbf{E}[\mathbf{X}])^T]$$

$$\sigma^2 = \text{Var}[\mathbf{X}] = \mathbf{E}[(\mathbf{X} - \mathbf{E}[\mathbf{X}])^2] = \mathbf{E}[(\mathbf{X} - \mu)^2]$$

$$\mu = \mathbf{E}[\mathbf{X}]$$

$$\text{Var}[X] = \text{Cov}[X] = E[(X - E[X])(X - E[X])^T]$$

$$\text{Cov}[X, Y] = E[(X - \mu)(Y - \nu)^T]$$

$$\sum = E[(X - E[X])(X - E[X])^T], \mu = E[X]$$

$$\sum = E[XX^T] - \mu\mu^T$$

$$\text{Var}[AX + a] = A \text{Var}[X] A^T$$

$$\text{Cov}[X, Y] = \text{Cov}[Y, X]$$

$$\text{Cov}[X_1 + X_2, Y] = \text{Cov}[X_1, Y] + \text{Cov}[X_2, Y]$$

$$\text{Var}[X + Y] = \text{Var}[X] + \text{Cov}[X, Y] + \text{Cov}[Y, X] + \text{Var}[Y]$$

$$\text{Cov}[AX, B^T Y] = A \text{Cov}[X, Y] B$$

X, X_1, X_2 : $p \times 1$ vector

Y : $q \times 1$ vector

a : $q \times 1$ vector

A, B : $q \times p$ matrices

$$c^T \sum = \text{Cov}[c^T X, X]$$

$$d^T \sum c = \text{Cov}[d^T X, c^T X]$$

$$\text{Var}[b^T X] = b^T \text{Var}[X] b$$

Matrix, Derivative, det, tr

$$\text{Inverse}\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right] = \begin{pmatrix} \frac{d}{-b c + a d} & -\frac{b}{-b c + a d} \\ -\frac{c}{-b c + a d} & \frac{a}{-b c + a d} \end{pmatrix}$$

$$\text{Inverse}\left[\begin{pmatrix} S_{1,1} & S_{1,2} \\ S_{2,1} & S_{2,2} \end{pmatrix}\right] = \begin{pmatrix} \frac{S_{2,2}}{-S_{1,2} S_{2,1} + S_{1,1} S_{2,2}} & -\frac{S_{1,2}}{-S_{1,2} S_{2,1} + S_{1,1} S_{2,2}} \\ -\frac{S_{2,1}}{-S_{1,2} S_{2,1} + S_{1,1} S_{2,2}} & \frac{S_{1,1}}{-S_{1,2} S_{2,1} + S_{1,1} S_{2,2}} \end{pmatrix}$$

$$\text{Det}\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right] = -b c + a d$$

$$\text{Tr}\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right] = a + d$$

$$\text{Det}[S] = \text{Det}\left[\begin{pmatrix} S_{1,1} & S_{1,2} \\ S_{2,1} & S_{2,2} \end{pmatrix}\right] = -S_{1,2} S_{2,1} + S_{1,1} S_{2,2}$$

$$\text{chol}[A] = R$$

$$R^T R = A$$

- The product of the eigenvalues is equal to the determinant of A

$$\det[A] = \prod_{i=1}^{N_\lambda} \lambda_i^{n_i}$$

- The sum of the eigenvalues is equal to the trace of A

$$\text{tr}[A] = \sum_{i=1}^{N_\lambda} n_i \lambda_i$$

■ Vector calculus

The tangent vector to a curve $x : \mathbb{R} \rightarrow \mathbb{R}^n$

$$\frac{\partial x}{\partial t} = \begin{pmatrix} \frac{\partial x_1}{\partial t} \\ \vdots \\ \frac{\partial x_n}{\partial t} \end{pmatrix}$$

The gradient of a scalar function $f : \mathbb{R}^n \rightarrow \mathbb{R}$

$$\frac{\partial f}{\partial x} = \left(\frac{\partial f}{\partial x_1} \quad \dots \quad \frac{\partial f}{\partial x_n} \right)$$

The directional derivative of f in the direction of v is then

$$\nabla_v f = \frac{\partial f}{\partial x} v$$

The pushforward of differential of a function $f : \mathbb{R}^m \rightarrow \mathbb{R}^n$ is described by the Jacobian matrix

$$\frac{\partial f}{\partial x} = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \dots & \frac{\partial f_1}{\partial x_m} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \dots & \frac{\partial f_n}{\partial x_m} \end{pmatrix}$$

The pushforward along f of a vector v in $\mathbb{R}^m$

$$d f (v) = \frac{\partial f}{\partial x} v$$

■ Matrix calculus

The tangent vector of a curve $F : \mathbb{R} \rightarrow M(n, m)$

$$\frac{\partial F}{\partial t} = \begin{pmatrix} \frac{\partial F_{1,1}}{\partial t} & \dots & \frac{\partial F_{1,m}}{\partial t} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_{n,1}}{\partial t} & \dots & \frac{\partial F_{n,m}}{\partial t} \end{pmatrix}$$

The gradient of a scalar function $f : M(n, m) \rightarrow \mathbb{R}$

$$\frac{\partial f}{\partial X} = \begin{pmatrix} \frac{\partial f}{\partial x_{1,1}} & \dots & \frac{\partial f}{\partial x_{1,m}} \\ \vdots & \ddots & \vdots \\ \frac{\partial f}{\partial x_{n,1}} & \dots & \frac{\partial f}{\partial x_{n,m}} \end{pmatrix}$$

$$\nabla_Y f = \text{tr} \left[\frac{\partial f}{\partial x} Y \right]$$

$$F : M(n, m) \rightarrow M(p, q)$$

$$\frac{\partial \mathbf{F}}{\partial \mathbf{X}} = \begin{pmatrix} \frac{\partial \mathbf{F}}{\partial x_{1,1}} & \cdots & \frac{\partial \mathbf{F}}{\partial x_{1,m}} \\ \vdots & \ddots & \vdots \\ \frac{\partial \mathbf{F}}{\partial x_{n,1}} & \cdots & \frac{\partial \mathbf{F}}{\partial x_{n,m}} \end{pmatrix}$$

$$d\mathbf{F}(\mathbf{Y}) = \text{tr} \left(\frac{\partial \mathbf{F}}{\partial \mathbf{X}} \mathbf{Y} \right)$$

$$\frac{\partial \mathbf{F}}{\partial \mathbf{X}} = \begin{pmatrix} \frac{\partial F_{1,1}}{\partial x} & \cdots & \frac{\partial F_{1,m}}{\partial x} \\ \vdots & \ddots & \vdots \\ \frac{\partial F_{n,1}}{\partial x} & \cdots & \frac{\partial F_{n,m}}{\partial x} \end{pmatrix}$$

$$\frac{\partial \mathbf{F}}{\partial \mathbf{X}} = \begin{pmatrix} \frac{\partial \mathbf{F}}{\partial x_{1,1}} & \cdots & \frac{\partial \mathbf{F}}{\partial x_{1,m}} \\ \vdots & \ddots & \vdots \\ \frac{\partial \mathbf{F}}{\partial x_{n,1}} & \cdots & \frac{\partial \mathbf{F}}{\partial x_{n,m}} \end{pmatrix}$$

Jacobian matrix

$$\mathbf{D} \mathbf{F}(\mathbf{x}) = \frac{\partial \text{vec } \mathbf{F}(\mathbf{X})}{\partial (\text{vec } \mathbf{X})^T}$$

■ Chain rule

$$\frac{\partial \mathbf{Z}}{\partial \mathbf{X}} = \frac{\partial \mathbf{Z}}{\partial \mathbf{Y}} \frac{\partial \mathbf{Y}}{\partial \mathbf{X}}$$

■ Product rule

$$\frac{\partial (\mathbf{Y}\mathbf{Z})}{\partial \mathbf{X}} = \frac{\partial \mathbf{Y}}{\partial \mathbf{X}} \mathbf{Z} + \mathbf{Y} \frac{\partial \mathbf{Z}}{\partial \mathbf{X}}$$

■ Derivative of linear functions

$$\frac{\partial \mathbf{a}^T \mathbf{x}}{\partial \mathbf{x}} = \frac{\partial \mathbf{x}^T \mathbf{a}}{\partial \mathbf{x}} = \mathbf{a}^T$$

$$\frac{\partial \mathbf{A}\mathbf{x}}{\partial \mathbf{x}} = \frac{\partial \mathbf{x}^T \mathbf{A}}{\partial \mathbf{x}} = \mathbf{A}$$

■ Derivative of quadratic functions

$$\frac{\partial \mathbf{x}^T \mathbf{A} \mathbf{x}}{\partial \mathbf{x}} = \mathbf{x}^T (\mathbf{A}^T + \mathbf{A}) = \mathbf{x}^T \mathbf{A}^T + \mathbf{x}^T \mathbf{A}$$

$$\frac{\partial (\mathbf{A}\mathbf{x} + \mathbf{b})^T \mathbf{C} (\mathbf{D}\mathbf{x} + \mathbf{e})}{\partial \mathbf{x}} = (\mathbf{D}\mathbf{x} + \mathbf{e})^T \mathbf{C}^T \mathbf{A} + (\mathbf{A}\mathbf{x} + \mathbf{b})^T \mathbf{C} \mathbf{D}$$

$$\frac{\partial \|\mathbf{x} - \mathbf{a}\|}{\partial \mathbf{x}} = \frac{(\mathbf{x} - \mathbf{a})^T}{\|\mathbf{x} - \mathbf{a}\|}$$

■ Derivative of matrix traces

$$\begin{aligned}\frac{\partial \text{tr}(AC)}{\partial A} &= C^T \\ \frac{\partial \text{tr}(AXB)}{\partial X} &= \frac{\partial \text{tr}(B^T X^T A^T)}{\partial X} = BA \\ \frac{\partial \text{tr}(ABA^T)}{\partial A} &= 2AB \\ \frac{\partial \text{tr}(AXBX^T C)}{\partial X} &= BX^T CA + B^T X^T A^T C^T\end{aligned}$$

■ Derivative of matrix determinant

$$\frac{\partial \det X}{\partial X} = \text{adj } X = \det X \cdot X^{-1}$$

Jacobian matrix

$$F : \mathbb{R}^n \rightarrow \mathbb{R}^m$$

$$J = \begin{pmatrix} \frac{\partial y_1}{\partial x_1} & \dots & \frac{\partial y_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial y_m}{\partial x_1} & \dots & \frac{\partial y_m}{\partial x_n} \end{pmatrix}$$

$$J_F(x_1, \dots, x_n) \text{ and } \frac{\partial (y_1, \dots, y_m)}{\partial (x_1, \dots, x_n)}$$

$$F(x) = F(p) + J_F(p)(x - p) + o(\|x - p\|)$$

■ ex1

$$x_1 = r \sin[\theta] \cos[\phi]$$

$$x_2 = r \sin[\theta] \sin[\phi]$$

$$x_3 = r \cos[\theta]$$

$$J_F(r, \theta, \phi) = \begin{pmatrix} \frac{\partial x_1}{\partial r} & \frac{\partial x_1}{\partial \theta} & \frac{\partial x_1}{\partial \phi} \\ \frac{\partial x_2}{\partial r} & \frac{\partial x_2}{\partial \theta} & \frac{\partial x_2}{\partial \phi} \\ \frac{\partial x_3}{\partial r} & \frac{\partial x_3}{\partial \theta} & \frac{\partial x_3}{\partial \phi} \end{pmatrix} = \begin{pmatrix} \sin[\theta] \cos[\phi] & r \cos[\theta] \cos[\phi] & -r \sin[\theta] \sin[\phi] \\ \sin[\theta] \sin[\phi] & r \cos[\theta] \sin[\phi] & r \sin[\theta] \cos[\phi] \\ \cos[\theta] & -r \sin[\theta] & 0 \end{pmatrix}$$

■ ex2

$$x = r \cos[\phi]$$

$$y = r \sin[\phi]$$

$$J_F(r, \phi) = \begin{pmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \phi} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \phi} \end{pmatrix} = \begin{pmatrix} \cos[\phi] & -r \sin[\phi] \\ \sin[\phi] & r \cos[\phi] \end{pmatrix}$$

$$\int \int_A dx dy = \int \int_B r dr d\phi$$

■ ex3

$$\mathbf{F} : \mathbb{R}^n \rightarrow \mathbb{R}^m$$

$$y_1 = F_1(x_1, \dots, x_n)$$

$$\vdots$$

$$y_m = F_m(x_1, \dots, x_n)$$

$$\mathbf{J}_F(x_1, \dots, x_n) = \begin{pmatrix} \frac{\partial y_1}{\partial x_1} & \dots & \frac{\partial y_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial y_m}{\partial x_1} & \vdots & \frac{\partial y_m}{\partial x_n} \end{pmatrix}$$

■ cholesky factorization

$$\mathbf{R} = \text{chol}(\mathbf{A})'$$

$$\mathbf{R} * \mathbf{R}' = \mathbf{A}$$

$$\mathbf{R} = \text{chol}(\mathbf{A})$$

$$\mathbf{R}' * \mathbf{R} = \mathbf{A}$$

Ex.

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{F}_k \hat{\mathbf{x}}_{k-1|k-1} + \mathbf{B}_k \mathbf{u}_k + \mathbf{w}_k$$

$$\mathbf{x}_k = \begin{pmatrix} \mathbf{x}_k \\ \dot{\mathbf{x}}_k \end{pmatrix}$$

$\mathbf{x}_k$: k시점에서의 위치

$\dot{\mathbf{x}}_k$: k시점에서의 속도

$\mathbf{a}_k$: 임의로 가해진 가속도

뉴턴의 운동법칙

$$\mathbf{x}_k = \mathbf{x}_{k-1} + \dot{\mathbf{x}}_{k-1} dt + \mathbf{a}_k \frac{dt^2}{2}$$

$$\dot{\mathbf{x}}_k = \dot{\mathbf{x}}_{k-1} + \mathbf{a}_k dt$$

$$\mathbf{x}_k = \mathbf{F} \mathbf{x}_{k-1} + \mathbf{w}_k$$

$$\mathbf{w}_k = \mathbf{B} \mathbf{a}_k$$

$$\mathbf{F} = \begin{pmatrix} \mathbf{1} & dt \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} \frac{dt^2}{2} \\ dt \end{pmatrix}$$

$$\mathbf{Q} = \text{Cov}[\mathbf{B} \mathbf{a}_k] = \mathbf{E}[(\mathbf{B} \mathbf{a}) (\mathbf{B} \mathbf{a})^T] = \mathbf{B} \mathbf{E}[\mathbf{a}^2] \mathbf{B}^T = \mathbf{B} [\sigma_a^2] \mathbf{B}^T = \sigma_a^2 \mathbf{B} \mathbf{B}^T = \begin{pmatrix} \frac{dt^4}{4} & \frac{dt^3}{2} \\ \frac{dt^3}{2} & dt^2 \end{pmatrix} \sigma_a^2$$

$$\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{v}_k$$

$$\mathbf{H} = (\mathbf{1} \quad \mathbf{0})$$

$$\mathbf{R} = \mathbf{E}[\mathbf{v}_k \mathbf{v}_k^T] = [\sigma_z^2]$$

$$\hat{\mathbf{x}}_{\theta|\theta} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \end{pmatrix}$$

$$\mathbf{P}_{\theta|\theta} = \begin{pmatrix} \mathbf{L} & \mathbf{0} \\ \mathbf{0} & \mathbf{L} \end{pmatrix}$$

http://ko.wikipedia.org/wiki/%EC%B9%BC%EB%A7%8C_%ED%95%84%ED%84%B0