

Matrix Matlab

■ Mean, Variance, Covariance

$$\mu = E[X]$$

$$\sigma^2 = \text{var}[X] = E[(X - E[X])^2]$$

$$X = [X_{11} \ X_{12} \ X_{13}; \ X_{21} \ X_{22} \ X_{23}; \ X_{31} \ X_{32} \ X_{33}]$$

$$X = \begin{pmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{pmatrix}$$

$$m = \text{mean}(X) = \frac{1}{n} \sum_{i=1}^n [X_{i1} \ X_{i2} \ \dots \ X_{in}]$$

$$\text{mean}(X) = \frac{1}{3} \sum_{i=1}^3 [X_{i1} \ X_{i2} \ X_{i3}] = \left[\frac{X_{11} + X_{21} + X_{31}}{3} \quad \frac{X_{12} + X_{22} + X_{32}}{3} \quad \frac{X_{13} + X_{23} + X_{33}}{3} \right]$$

■ $\text{var}(X,1) = \text{mean}(v.^2)$

$$v = X - \text{ones}(3, 1) * \text{mean}(X)$$

$$\text{var}(X, 1) = \text{mean}(v.^2)$$

■ $\text{cov}(X) = \frac{v' * v}{n - 1}$

$$[n, p] = \text{size}(X)$$

$$v = X - \text{ones}(3, 1) * \text{mean}(X)$$

$$\text{cov}(X) = \frac{v' * v}{n - 1}$$

$$\text{var}(x) = \text{diag}(\text{cov}(x))'$$

Matrix Mathematica

$$X = \{-1, 4, -9, -0, 3, 3, 9, -2, 5, 5\};$$

$$\text{Mean}[X] = \frac{\text{Total}[X]}{\text{Length}[X]}$$

$$\text{Variance}[X] = \frac{\text{Total}[(X - \text{Mean}[X])^2]}{\text{Length}[X] - 1}$$

$$\text{StandardDeviation}[X] = \text{Sqrt}[\text{Variance}[X]]$$

■ Mean

$$X = \begin{pmatrix} -1 & 4 & -9 \\ 0 & -3 & 3 \\ 9 & -2 & 5 \end{pmatrix};$$

Mean[X]

$$\begin{pmatrix} \frac{8}{3} \\ -\frac{1}{3} \\ -\frac{1}{3} \end{pmatrix}$$

■ Covariance

$$V = (X - \{\text{Mean}[X], \text{Mean}[X], \text{Mean}[X]\})$$

$$\frac{V^T \cdot V}{\text{Length}[X] - 1}$$

$$\begin{pmatrix} \frac{91}{3} & -\frac{29}{3} & \frac{85}{3} \\ -\frac{29}{3} & \frac{43}{3} & -\frac{83}{3} \\ \frac{85}{3} & -\frac{83}{3} & \frac{172}{3} \end{pmatrix}$$

■ Covariance to Variance : var(x)

$$V = (X - \{\text{Mean}[X], \text{Mean}[X], \text{Mean}[X]\})$$

$$\text{Total}\left[\frac{V^T \cdot V}{\text{Length}[X] - 1} * \text{IdentityMatrix}[3]\right]$$

Variance[X]

$$\left\{\frac{91}{3}, \frac{43}{3}, \frac{172}{3}\right\}$$

$$\text{Mean}[(X - \{\text{Mean}[X], \text{Mean}[X], \text{Mean}[X]\})^2] * \frac{\text{Length}[X]}{\text{Length}[X] - 1}$$

■ Variance : var(x,1)

$$\text{Mean}[(X - \{\text{Mean}[X], \text{Mean}[X], \text{Mean}[X]\})^2]$$

{20.22,9.55,38.22}

■ IdentityMatrix[3]

■ DiagonalMatrix[{a,b,c,d}]

X[[1]]

{-1,4,-9}

`X[[All, 1]]`

{-1,0,9}

■ Inverse, Determinant, Trace, Transpose

$$\text{Inverse}\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right] = \begin{pmatrix} \frac{d}{-b c + a d} & -\frac{b}{-b c + a d} \\ -\frac{c}{-b c + a d} & \frac{a}{-b c + a d} \end{pmatrix}$$

$$\text{Det}\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right] = -b c + a d$$

$$\text{Tr}\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right] = a + d$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^T = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$$

$$\text{Mean}\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right] = \begin{pmatrix} \frac{1}{2} (a + c) \\ \frac{1}{2} (b + d) \end{pmatrix}$$

$$\text{Variance}\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right] = \begin{pmatrix} \frac{1}{2} (a - c)^2 \\ \frac{1}{2} (b - d)^2 \end{pmatrix}$$

$$\text{Covariance}\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right] = \begin{pmatrix} \frac{1}{2} (a - c)^2 & \frac{1}{2} (a - c) (b - d) \\ \frac{1}{2} (a - c) (b - d) & \frac{1}{2} (b - d)^2 \end{pmatrix}$$

■ How to Covariance

$$X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

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Covariance[X_] :=
{
  V = (X - {Mean[X], Mean[X]});
  R =  $\frac{V^T \cdot V}{\text{Length}[X] - 1}$  // Simplify // MatrixForm
}[[1]];
```

eigenvalue & eigenvector

eigenvalue : λ

eigenvector : V_λ

$$A V_\lambda = \lambda V_\lambda$$

$$A = V_\lambda \lambda V_\lambda^{-1}$$

$$(A - \lambda \text{Eye}) V_\lambda = 0$$

$$\text{Det}[A - \lambda \text{Eye}] = 0$$

■ 2×2 Matrix

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$\text{Eye} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\text{Det} \left[\begin{pmatrix} a - \lambda & b \\ c & d - \lambda \end{pmatrix} \right] = (a - \lambda)(d - \lambda) - bc = \lambda^2 - (a + d)\lambda + (ad - bc)$$

$$\text{Solve}[\text{Det}[A - \lambda \text{Eye}] = 0, \lambda][[1]][[1]][[2]]$$

$$\lambda = \frac{1}{2} \left(a + d \pm \sqrt{4bc + (a - d)^2} \right)$$

$$\text{Tr}[A] = a + d$$

$$\text{Det}[A] = ad - bc$$

$$\text{Det}[A - \lambda \text{Eye}] = \lambda^2 - \text{Tr}[A]\lambda + \text{Det}[A]$$

$$\begin{aligned} \lambda &= \frac{1}{2} \left((a + d) \pm \sqrt{(a + d)^2 - 4(ad - bc)} \right) \\ &= \frac{1}{2} \left(\text{Tr}[A] \pm \sqrt{\text{Tr}[A]^2 - 4 \text{Det}[A]} \right) \end{aligned}$$

■ Matlab

$$[V, D] = \text{eig}(A)$$

$$A = V * D * \text{inv}(V)$$

Similarity Transformation Diagonalizable

$$A V_1 = \lambda_1 V_1, \dots, A V_n = \lambda_n V_n$$

$$C = [V_1, \dots, V_n]$$

(1×n) Vector

$$\begin{aligned} A C &= A [V_1, \dots, V_n] = [A V_1, \dots, A V_n] = [\lambda_1 V_1, \dots, \lambda_n V_n] \\ &= [V_1, \dots, V_n] \begin{pmatrix} \lambda_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \lambda_n \end{pmatrix} = C \begin{pmatrix} \lambda_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \lambda_n \end{pmatrix} \end{aligned}$$

$$C^{-1} A C = \begin{pmatrix} \lambda_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \lambda_n \end{pmatrix}$$

if C is orthogonal (직교)
then $C^{-1} = C^T$

$$S = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

$$S^{-1} = \text{Inverse}[S] = \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}$$

$$A = \begin{pmatrix} -1 & 4 \\ 0 & 3 \end{pmatrix}$$

$$S^{-1} A S = \text{Inverse}[S] . A . S = \begin{pmatrix} 3 & 0 \\ 0 & -1 \end{pmatrix}$$